The fundamental role of the Oort cloud in determining the flux of comets through the planetary system

V.V. Emel'yanenko[1], D.J. Asher[2] and M.E. Bailey[2]
[1] *South Ural University, Chelyabinsk, 454080, Russia*
[2] *Armagh Observatory, College Hill, Armagh, BT61 9DG*

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ABSTRACT

A model of the Oort cloud has been developed by accounting for planetary, stellar and Galactic perturbations using numerical symplectic integrations covering 4.5 Gyr. The model is consistent with the broad dynamical characteristics of the observed cometary populations injected from the Oort cloud into different regions of the Solar system. We show that the majority of observed high-eccentricity trans-Neptunian objects, Centaurs and short-period comets have visited the Oort cloud ($a > 1000$ au) during their dynamical history. Assuming from observations that the near-parabolic flux from the Oort cloud with absolute magnitudes $H_{10} < 7$, perihelion distances $q < 5$ au and $a > 10^4$ au is approximately 1 comet per year, our calculations imply a present Oort cloud population of $\sim 5 \times 10^{11}$ comets with $H_{10} < 10.9$. Roughly half this number has $a > 10^4$ au. The number of comets reaching the planetary region from the Oort cloud ($a > 1000$ au) is more than an order of magnitude higher per unit perihelion distance immediately beyond Neptune than in the observable zone $q < 5$ au. Similarly, the new-comet flux from the Oort cloud per unit perihelion distance is a few tens of times higher in the near-Neptune region than in the observable zone. The present number of high-eccentricity trans-Neptunian objects ($q > 30$ au and $60 < a < 1000$ au) originating from the Oort cloud is in the approximate range 1–3×10^{10}, depending on details of the initial model. A substantial fraction of these have $a > 200$ au and/or $q > 40$ au, and they are found mostly to originate from initial orbits with $25 < q < 36$ au. Similarly, the number of Centaurs produced from the Oort cloud, where we define Centaurs to have $5 < q < 28$ au and $a < 1000$ au, is smaller by a factor of 20–30. About 90 per cent of these Centaurs have $a > 60$ au. Objects that have visited the Oort cloud represent a substantial fraction of the Jupiter-family comet population, achieving short-period orbits by a process of gradual dynamical transfer, including a Centaur stage, from the outer Solar system to near-Earth space. A similar mechanism produces Halley-type comets, in addition to the well-known diffusion process operating at small perihelion distances.

Key words: celestial mechanics – comets: general – Edgeworth-Kuiper belt, Oort cloud – minor planets, asteroids – Solar system: formation.

1 INTRODUCTION

Oort's (1950) proposal that the Solar system is surrounded by a vast, nearly spherical swarm of comets with a nearly isotropic distribution of orbital velocities (at least in the cloud's outer regions) is widely accepted. The Oort cloud is the proximate source of observed nearly parabolic, so-called 'new' comets entering the planetary region, and is also the presumed source of the long-period comet flux and the majority of Halley-type comets (HTCs). It is also widely assumed that the comets which make up the Oort cloud must have formed much closer to the planetary system, probably around the time of planet formation, within 1–50 Myr of the formation of the Sun.

What is less widely accepted, and indeed less widely understood, is the precise proportion of Halley-type comets that originate in the Oort cloud; and similarly, the proportions of other well-known classes of objects in the planetary region. Such bodies include, for example, long-period and Halley-type cometary asteroids and 'Damocloids', Jupiter-family short-period comets, Centaurs, and high-eccentricity trans-Neptunian objects (TNOs). However, since the late 1990s, the majority of researchers have accepted that the

Edgeworth-Kuiper belt (EKB) or the Kuiper belt is the dominant source of Centaurs and observed Jupiter-family comets (JFCs). Indeed, some authors have argued that this flattened source region may even produce the majority of HTCs (Levison et al. 2006).

However, as work to understand the observational properties and theoretical structures of the EKB has proceeded, it is remarkable that even the best current theories of this region have often failed correctly to predict the next observational 'surprise'. The outer Solar system is now known to comprise a very complicated distribution of so-called 'small' bodies. These range in heliocentric distance from the resonant and non-resonant structures found in the 'classical' Kuiper belt or EKB just beyond Neptune, to 'scattered disc objects' with perihelion distances close to Neptune and moderately high-eccentricity orbits, and even to objects that appear to have no obvious dynamical connection with the present planetary system. In some cases, e.g. the Centaur (127546) 2002 XU_{93} and the dwarf planet (131699) Eris (formerly 2003 UB_{313}), these 'scattered disc objects' have high inclinations, seemingly incompatible with an origin in a flattened, protoplanetary disc.

The orbital properties of long period and high inclination are, of course, among the principal characteristics usually associated with the Oort cloud. It is reasonable, therefore, to suppose that these unusual objects may originate in the Oort cloud rather than in the primordial EKB. Similarly, despite the prevailing view that Centaurs and JFCs predominantly originate in the EKB or Scattered Disc, it is important to ask whether a substantial fraction of these classes of object may also originate in the Oort cloud.

This paper therefore develops the idea that the Oort cloud is the proximate source of the *majority* of comets passing through the inner Solar system. In particular, we draw together the results of our own dynamical investigations, and those of others, which show in the context of a simple 'standard' model for the formation and evolution of the comet cloud, that many observational results, including those pertaining to HTCs, JFCs and Centaurs, can be understood without appeal to a flattened EKB source. Our thesis has the merit of unifying a very wide range of observational material.

Our results show that the Oort cloud plays a fundamental role in determining the flux of comets through the planetary system. Indeed, we find that even some TNOs (e.g. the high-eccentricity population) can be explained as a consequence of the dynamical evolution of objects from the Oort cloud. We conclude that the wider unsolved problem, namely the origin of comets, should focus on the formation, structure and evolution of the Oort cloud.

We emphasize that we do not seek to eliminate any role for the EKB. It is observed to exist, and a wide range of theoretical investigations indicate that it is a potentially significant source of relatively short-period, predominantly low-inclination objects (e.g. observed Centaurs and JFCs). We argue, however, that the EKB is of secondary importance relative to the Oort cloud. Thus, there should be at least two different types of comet: for example, those that formed in the primordial region which led to the observed Oort cloud (i.e. possibly, but not necessarily, the protoplanetary disc), and those that formed farther out in a region containing rather more stable orbits. The latter would be somewhat decoupled from the observed planetary system and could be associated with the classical Edgeworth-Kuiper belt.

In this paper we set up a model to represent the Oort cloud (Section 2), and then evaluate the resulting Oort cloud's contribution to the high-eccentricity cometary flux through the planetary region (Section 3), to TNOs and Centaurs (Section 4), and to short-period Jupiter-family and Halley-type comets (Section 5).

2 MODEL

For this work, we first require a model of the Oort cloud. To create this, we assume that the Oort cloud has been produced as a by-product of planet formation. We note, however, that this is not an essential element of the model. Indeed, from some points of view, it may be quite unlikely, unless cometary masses are much smaller than currently accepted, but it is our working hypothesis. Our objective is to use a realistic Oort cloud model to study the cometary populations entering the planetary system from the Oort cloud, not to study different models for the formation and early evolution of the Solar system or how the Oort cloud itself may have originated. Thus, we assume as a working hypothesis that the eight planets were formed as a result of the accretion of planetesimals in the protoplanetary disc, and that after the period of planet formation was essentially finished the outcome was an initial distribution of cometary orbits with given semi-major axes, eccentricities and inclinations. Following Duncan, Quinn & Tremaine (1987), we choose initial conditions that represent a plausible stage in the process of creating an Oort cloud within this planetesimal picture.

This initial distribution of orbits is then evolved for the age of the Solar system in a 'steady-state' dynamical model, i.e. one that corresponds approximately to the Sun's present situation in the Galaxy for 4.5 Gyr. Of course, we recognize that the first few tens of Myr may have been quite different (e.g. involving significant changes in the orbits, and perhaps even the number, of planets), and that during the first few hundred Myr the Solar system may have been subject to a very different Galactic environment from the one currently experienced. However, in order to develop a simple but reasonably realistic model of the Oort cloud, we ignore such complications. Similarly, we ignore the complicating effects of occasional strong perturbations by material in Galactic spiral arms, such as giant molecular clouds (GMCs) and their hierarchical substructure, which could significantly perturb the outer regions of the Oort cloud, possibly at mean intervals as short as 500 Myr, and other perturbers. Such matters are left as topics for future research.

2.1 Initial conditions

Initial semi-major axes of test particles were uniformly distributed in the range $50 < a < 300$ au, and initial inclinations were distributed according to a 'sine law' scaled to the interval $0 < i < 40°$, so as to have a mean inclination $i = 20°$ (i.e. probability density proportional to $\sin(4.5\,i)$ in this interval). Perihelion distances were uniformly distributed: 167 particles with $5 < q < 10$ au, 500 with $10 < q < 25$ au, and 367 with $25 < q < 36$ au, giving a total of 1034 particles at

Table 1. Evolution of comets from the Oort cloud. 'Maximum' is the number of Oort cloud clones that would have been generated if all original particles had reached the Oort cloud region, the relative numbers in each region of initial q simply being due to our decision to integrate particles distributed uniformly in q. The initial number in the Oort cloud depends on the number of particles that actually reached the Oort cloud region during 4.5 Gyr. The remaining lines show the numbers of clones from the Oort cloud that survive and are in the (a, q) regions indicated, at $t = 4.5$ Gyr.

Initial q (au) at $t = 0$ (4.5 Gyr ago)	5-10	10–25	25–36	5–36
Maximum no. of clones	8350	25000	18350	51700
Initial no. of clones in Oort cloud ($10^3 < a < 10^5$ au, various t)	5900	21900	13850	41650
$t = 4.5$ Gyr, $a > 10^3$ au	60	3758	5107	8925
$t = 4.5$ Gyr, $a > 10^4$ au	57	2602	2182	4841
$t = 4.5$ Gyr, $60 < a < 1000$ au, $q > 30$ au	0	19	309	328
$t = 4.5$ Gyr, $60 < a < 1000$ au, $28 < q < 35.5$ au	0	8	93	101

the start of the integration. These initial conditions may be compared with those of other authors who have investigated a broadly similar model, for example the pioneering work of Duncan et al. (1987) and the widely cited preprint of Dones et al. (2004a; also described by Dones et al. 2004b). Our initial conditions are 'warmer' than the 'warm' model of Dones et al. (2004a), but not as 'hot' as those chosen by Duncan et al. (1987).

Nevertheless, we emphasize that the results for these sorts of model do not depend sensitively on the choice of initial conditions provided that (a) the initial semi-major axes are chosen to be sufficiently small; (b) the initial ensemble includes orbits with perihelion distances distributed throughout the planetary system; and (c) the initial orbital inclination distribution is flattened, consistent with an origin for comets in the protoplanetary disc.

Models for different distributions of initial orbits throughout the planetary system can be constructed from our results by a suitable weighting of the results from the three ranges of q. Of more importance for the present work, however, is that essentially all memory of the initial conditions is lost (so far as the structure of the outer Oort cloud is concerned) after 4.5 Gyr of dynamical evolution under the combined influence of planetary and stellar perturbations, and the effects of the Galactic tide. The resulting model for the present-day Oort cloud is thus insensitive to the particular initial conditions that we have considered, although factors such as the efficiency of Oort cloud formation and the precise ratio of the number of comets in the inner and outer Oort clouds may depend on such details.

For example (cf. Section 3), whereas Duncan et al. (1987) find that their model Oort cloud has roughly four times as many comets with $a < 20000$ au as with $a > 20000$ au, our results for different initial distributions of orbits versus perihelion distance provide a corresponding ratio between two and four, the lower figure corresponding to the model ($\sim 1/q^2$) in which much more weight is given to initial orbits in the Jupiter-Saturn zone than in the Uranus-Neptune region. In general, Section 3 shows that the total number of comets in the Oort cloud and the ratio of the numbers in the inner and outer clouds (defined as $10^3 < a < 10^4$ au and $10^4 < a < 10^5$ au) is quite insensitive to changes in the initial distribution of orbits through the planetary system. Nevertheless, insofar as there are slight differences between the results for different models, observations can be used to discriminate between the different kinds of model.

2.2 Numerical procedure

The orbits of the initial test particles were integrated for 4.5 Gyr in a model Solar system containing the Sun and the four outer planets Jupiter through to Neptune. The masses of the terrestrial planets were added to the Sun, and the motion of the outer planets (with their present masses) was calculated on the basis of the secular perturbation theory of Brouwer & van Woerkom (1950) and Sharaf & Budnikova (1967). The dynamical evolution of the test particles was then calculated using the symplectic integrator described by Emel'yanenko (2002a) and Emel'yanenko, Asher & Bailey (2003). The method solves the Hamiltonian equations of barycentric motion for comets in the field of the Sun and planets. The adaptive time-step for this algorithm depends on the distance r from the centre and on the magnitude of perturbations. The integrator can therefore handle both high-eccentricity orbits and close encounters with planets. For small distances, the time-step is almost proportional to r in the absence of close encounters. For the majority of calculations, the adopted time-step was 15 days at $r = 5$ au and did not exceed 900 days for any distance.

The model for the Galaxy follows that of Byl (1986), except that the key Galactic parameters were chosen to correspond to those adopted by Levison, Dones & Duncan (2001), namely the Sun's angular speed $\Omega_0 = 26\,\mathrm{km\ s^{-1}\,kpc^{-1}}$ and the mid-plane density, ρ_0, of the Galactic disc in the Solar neighbourhood $\rho_0 = 0.1\,M_\odot\ \mathrm{pc}^{-3}$. Stellar perturbations were modelled following the procedure of Heisler, Tremaine & Alcock (1987).

In order not to waste time integrating objects on hyperbolic orbits or on the outskirts of the Oort cloud (from where they are highly unlikely to return to the planetary system), we considered all objects with $a > 10^5$ au (or strictly, $a^{-1} < 10^{-5}\,\mathrm{au}^{-1}$, as $a < 0$ is possible) as being immediately 'lost' from the system. We also stopped the integrations when $q < 2.5$ au, as such objects are unlikely to survive in the Oort cloud in the long term.

To save computer time we had just 1034 particles initially, but to ensure the final results provide a statistically significant sample we employed the following cloning procedure. Thus, whenever an object first evolved to large semi-major axis (we chose $a > 1000$ au) it was immediately cloned into 50 objects. The clones were produced by adding a very small change, Δa, to the semi-major axis of the object when it first achieved this condition, which was evaluated in barycentric coordinates at a sufficiently large distance from the Sun. For simplicity, Δa was chosen to be a uniformly distributed random variable in the range $0 < \Delta a < 3$ au, where the value of 3 au is arbitrary, provided it is sufficiently small. The main point about this cloning procedure is that it ensures that different clones return to the planetary system with essentially unchanged $1/a$-values but at totally different times so far as the relative configuration of the planets and their corresponding perturbations are concerned. Clones were integrated for the remainder of the 4.5 Gyr time frame, or, as with the original particles, until $a^{-1} < 10^{-5}$ au^{-1} or $q < 2.5$ au. It is possible for clones to return to the trans-Neptunian region $a < 1000$ au, and our primary interest is in such objects that come from the Oort cloud region $a > 1000$ au. This work does not consider the distribution of 'primordial' TNOs that have never entered the Oort cloud (although many such objects could exist in long-lived, relatively stable orbits), hence our choice of which particles to clone.

2.3 Preliminary features of results

Of the 1034 initial orbits, 833 reached the Oort cloud region (defined as $a > 1000$ au in this paper) within 4.5 Gyr (118 out of the 167 with initial perihelion distances in the range 5–10 au; 438 out of 500 with q in the range 10–25 au; and 277 out of 367 with q in the range 25–36 au). Cloning of these objects thus led to an initial ensemble of 41650 objects that had reached $a > 1000$ at some time during 4.5 Gyr. The majority of the remaining 201 particles were ejected from the Solar system in a single orbit (i.e. evolved during a single revolution from $a < 1000$ au to $a > 10^5$ au or evolved to $q < 2.5$ au), only one surviving for 4.5 Gyr on long-lived stable orbits from each of the initial ranges of perihelion distances 5–10 and 10–25 au, and 22 surviving from the initial range 25–36 au.

Roughly 80 per cent of the 41650 particles were ejected before the end of the integration, leaving a residual population (all a-values) after 4.5 Gyr comprising 60 clones originating from the region $5 < q < 10$ au, 3778 from the range $10 < q < 25$ au, and 5437 from $25 < q < 36$ au, a total of 9275 particles. Table 1 shows that at $t = 4.5$ Gyr most but not all of these particles are still in the Oort cloud ($a > 10^3$ au). It also shows that objects with initial q in the near-Neptune region ($25 < q < 36$ au) have the largest probability of being transferred into the Oort cloud and surviving until the present day. This probability is small for objects with initial q in the Jupiter-Saturn zone ($5 < q < 10$ au) as these planets are more likely to eject small bodies from the Solar system completely (cf. Safronov 1972), with the overwhelming majority of the few objects placed in the Oort cloud from this region finally residing in the outer cloud.

Figs 1 and 2 show orbital distributions for objects surviving after 4.5 Gyr. The $\cos i$ plot shows that at $a \lesssim 6000$ au the Oort cloud is still strongly flattened but by $a \gtrsim 8000$ au external perturbations have made it reasonably isotropic.

Fig. 3 provides a density profile in heliocentric distance r. There is no single power law $r^{-\gamma}$ that fits perfectly over the whole r range, but for reference we note best-fitting values in the following r ranges (in au): $\gamma = 2.8$ in $10^3 < r < 10^5$, $\gamma = 3.8$ in $10^4 < r < 10^5$ and $\gamma = 2.9$ in $3000 < r < 50000$. Duncan et al. (1987) found $\gamma = 3.5 \pm 0.5$ in $3000 < r < 50000$ and Dones et al. (2004b) give $\gamma \sim 3$ in $2000 < r < 200,000$. The results in Fig. 3 give equal weight to all clones; smaller weights for particles originating from larger q (cf. Section 3) would slightly increase the population at larger distances (Table 1) tending to make γ slightly smaller. Nevertheless, the broad agreement of γ with other authors' models, and also the overall results being rather insensitive to initial conditions (Duncan et al. 1987, p. 1335; Dones et al. 2004b, p. 166), support our claim that we have a realistic model for studying objects coming from the Oort cloud.

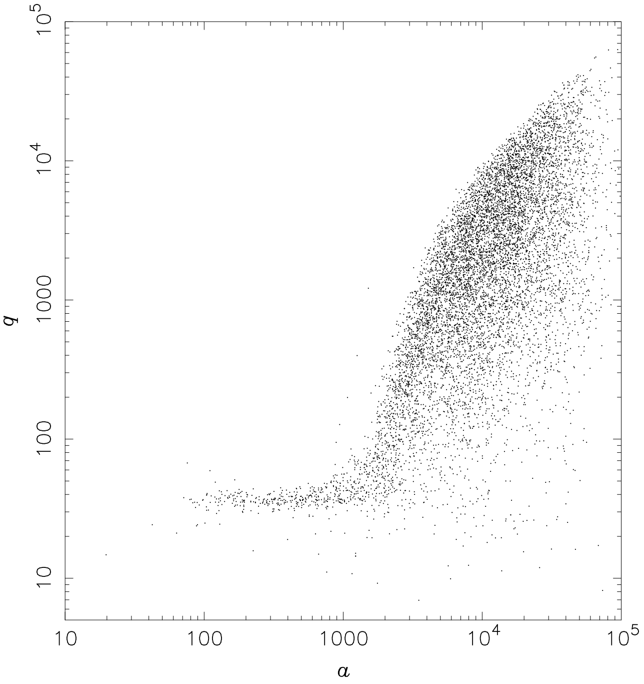

Figure 1. The distribution of a and q at 4.5 Gyr for objects that are or have been in the Oort cloud and are still surviving at this time.

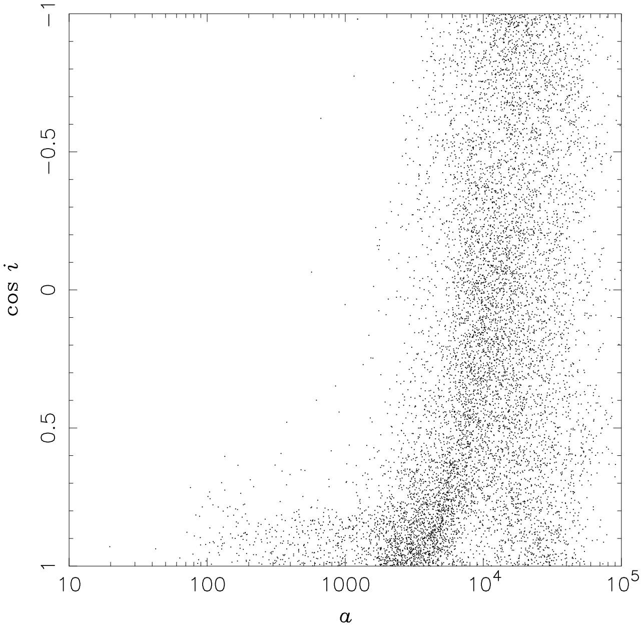

Figure 2. The distribution of a and $\cos i$ at 4.5 Gyr for objects that are or have been in the Oort cloud and are still surviving at this time.

Table 2. The number $N_{\rm OC}$ of Oort cloud comets at the present epoch for models defined by different initial distributions of objects versus perihelion distance in the early Solar system. $N_{\rm inner}$ and $N_{\rm outer}$ denote the numbers with semi-major axes in the ranges $10^3 < a < 10^4$ au and $10^4 < a < 10^5$ au respectively.

Initial q-distribution	$N_{\rm OC}$	$N_{\rm inner}$	$N_{\rm outer}$
Constant for $25<q<36$ au and zero otherwise	5.3×10^{11}	3.0×10^{11}	2.2×10^{11}
Constant for $5<q<36$ au and zero otherwise	4.8×10^{11}	2.2×10^{11}	2.6×10^{11}
$\sim 1/q$ for $5<q<36$ au and zero otherwise	4.6×10^{11}	1.9×10^{11}	2.7×10^{11}
$\sim 1/q^2$ for $5<q<36$ au and zero otherwise	4.5×10^{11}	1.6×10^{11}	2.9×10^{11}

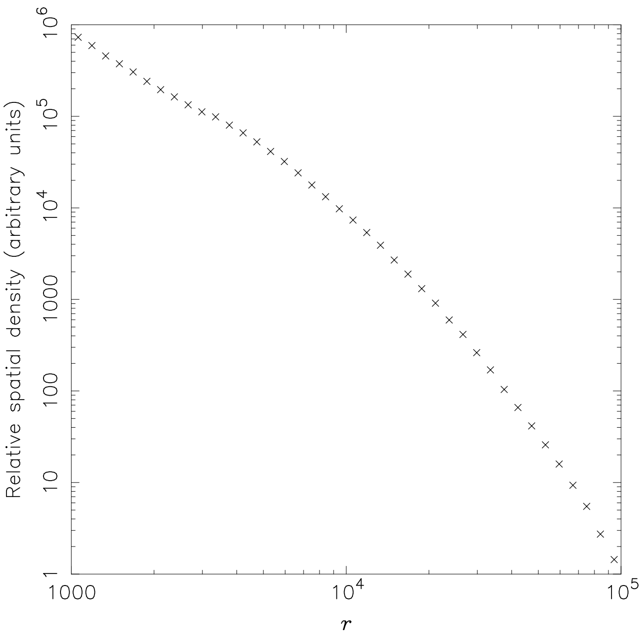

Figure 3. Variation of spatial density of Oort cloud comets with heliocentric distance. At smaller r where orbits are not isotropic (i.e. $r \lesssim 10000$ au) the spatial density is averaged over shells at given r rather than being a true density.

3 DISTRIBUTION OF NEAR-PARABOLIC COMETS IN THE PLANETARY REGION AND SIZE OF INNER AND OUTER OORT CLOUDS

Our computations allow us to estimate the number of objects in the Oort cloud by comparison of our results with the observed number of 'new' comets in near-Earth space. For the latter we adopt a value of 0.2 $\rm au^{-1}\ yr^{-1}$ for comets with absolute magnitudes $H_{10} < 7$ (Bailey & Stagg 1988). Although there are some uncertainties, different estimates are close to this value (Fernández 1982; Fernández & Gallardo 1999). Furthermore, we define objects of cometary size as having $H_{10} < 10.9$ (cf. Levison et al. 2002). The number of comets with $H_{10} < 10.9$ is approximately ten times larger than the number with $H_{10} < 7$ if we adopt a slope of $\alpha = 0.28$ for the cumulative distribution of comets versus H_{10} (proportional to $10^{\alpha H_{10}}$; Weissman & Lowry 2001). Thus, for the estimates in this paper, we assume an integrated new-comet flux with $q < 5$ au and $a > 10^4$ au of 10 objects of cometary size per year, corresponding to ~1 object per year with $H_{10} < 7$ and the same ranges of a and q.

We registered 263 objects reaching the region $q < 5$ au with $a > 10^4$ au during the last billion years of our integrations. However, many of these were injected into this region as a result of close star passages. Since there are no indications that such events have occurred close to the present epoch, the flux of dynamically 'new' comets averaged over 1 Gyr is probably larger than that observed at present.

In order to quantify this effect, we therefore made additional investigations. We took the surviving objects after 4.5 Gyr, cloned them 20 times and integrated these particles for a 200 Myr interval in which there were no close star passages. In order to weaken any fluctuations connected with the initial conditions for these objects we analysed results on the interval 50–200 Myr. We registered 560 particles reaching the region $q < 5$ au with $a > 10^4$ au in this 150 million year interval (3 from initial orbits having $5 < q < 10$ au, 266 from initial orbits having $10 < q < 25$ au, and 291 from initial orbits having $25 < q < 36$), giving an injection rate of new comets approximately 1.4 times less than that obtained by averaging our results over the 1 Gyr interval.

Table 2 provides results for the numbers of comets in the Oort cloud for different models, based on the 50–200 Myr data. The first line presents results for a model in which the present-epoch Oort cloud orbits were initially (4.5 Gyr ago) located only in the outer planetary region $25 < q < 36$ au. The second line gives data for a model in which the distribution of initial q is uniform in the range 5–36 au. The last two lines correspond to initial perihelion distances distributed according to q^{-1} and q^{-2}, i.e. with relatively fewer comets having initial perihelion distances in the Uranus-Neptune region. The Table shows the number of objects in the outer Oort cloud ($a > 10^4$ au), the number in the inner cloud ($10^3 < a < 10^4$ au), and the total number in the Oort cloud ($a > 10^3$ au).

Table 2 also shows that the total number of comets in the Oort cloud at the present epoch does not sensitively depend on the model chosen; and furthermore that the number of objects in the inner cloud is comparable to that in the outer part. This raises an important question, namely whether the Oort cloud can survive for the age of the Solar system when the additional perturbations of external molecular clouds are included, but this issue is beyond the scope of the present investigation.

Nevertheless, it is important to recognize the uncertainties underlying these estimates. For example, we do not know the real situation concerning recent close star passages through the Oort cloud (such close passages allowing a smaller number $N_{\rm OC}$ to produce the observed near-parabolic flux, which calibrates the model). Thus, the values in Table 2 could be made somewhat smaller (e.g. by a factor 1.4 if our model in the last 1 Gyr based on an initial uniform q-distribution is representative of the Sun's real environment at the present epoch). Similarly, Duncan et al.

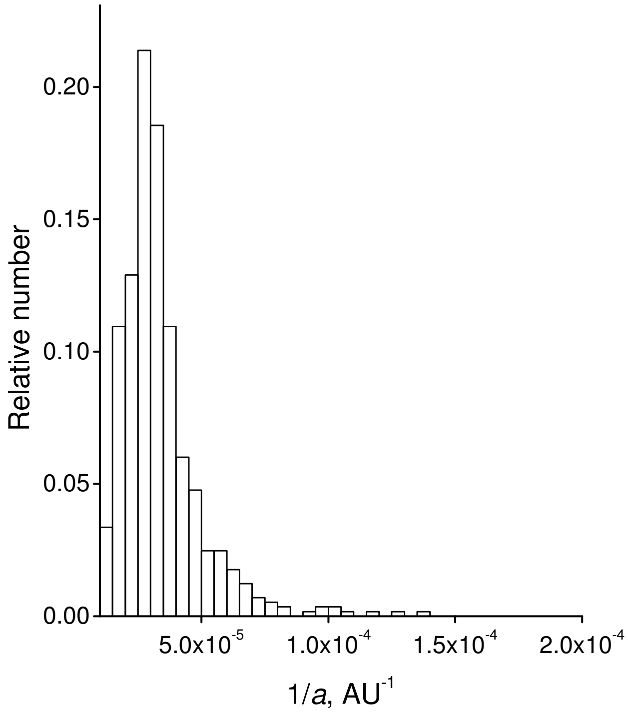

Figure 4. The relative number versus $1/a$ of dynamically new objects with $q < 5$ au (i.e. objects first entering the region $q < 5$ au).

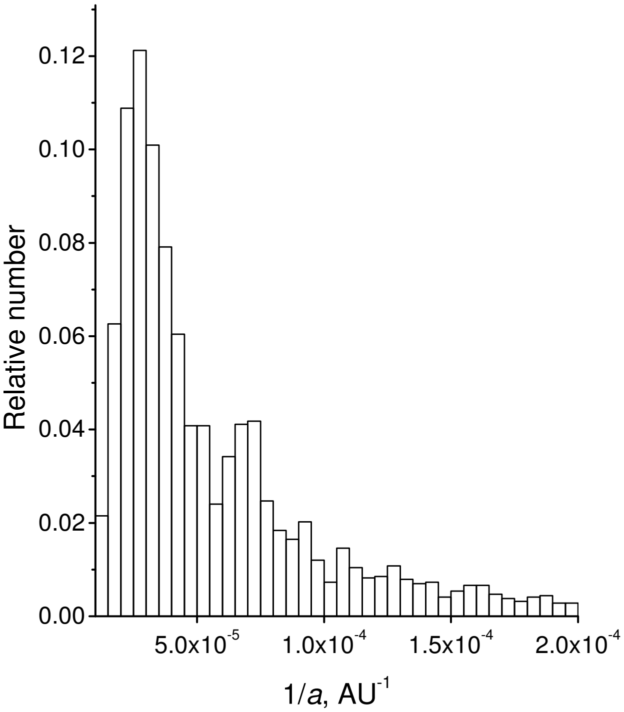

Figure 5. The relative number versus $1/a$ of dynamically new objects with $q < 35$ au (i.e. objects first entering the region $q < 35$ au).

(1987) found four times as many comets with $a < 20000$ au as with $a > 20000$ au. In the Dones et al. (2004a,b) model this ratio was close to unity, and from our integrations the same ratio is between two and four depending on the relative importance of different perihelion distances in the initial source region. Finally, the observed new-comet flux used to calibrate the model is itself rather uncertain.

Fig. 4 shows results, taken from the interval 50–200 Myr of our additional integrations, for the total number of comets visiting the region $q < 5$ au for the first time. Fig. 5 shows the corresponding distribution for the total number of comets visiting the region $q < 35$ au for the first time. We note the concentration of such small-q orbits towards large semi-major axes, which reinforces the usual explanation for the well-known Oort spike in the frequency distribution of observed $1/a$ values. Although observed comets with $a > 10^4$ au are often regarded as dynamically 'new', such objects can visit the planetary region several times during their previous evolution (Bailey 1977; Dybczyński 2001). We also note that Fig. 4 includes many objects that have experienced previous passages through the planetary system ($5 < q < 35$ au) during their orbital evolution. The extended 'tail' of objects towards larger $1/a$-values shown in Fig. 5 illustrates the existence of a significant inner Oort cloud flux through the outer planetary region.

Fig. 6 shows the relative number versus perihelion distance of objects with $a > 1000$ au, visiting the region $q < 35$ au for the first time. In particular, we find that the number of such dynamically new Oort cloud objects immediately beyond Neptune is more than an order of magnitude greater than that in the observable region $q < 5$ au.

It is also interesting to compare our results with the very comprehensive investigation of Wiegert & Tremaine (1999), which was aimed primarily at predicting the observed orbital distribution of new comets in order to provide constraints on the well-known problem of cometary fading (e.g. Bailey 1984, 2002; Levison et al. 2002). For example, although their model neglected the effects of passing stars and adopted initial inner and outer edges for the Oort cloud at $a = 10^4$ au and 5×10^4 au, the distribution of original $1/a$-values for new comets with $q < 3$ au shown in their fig. 11 is rather similar to our Fig. 4, both showing a sharp decrease for $1/a \gtrsim 4 \times 10^{-5}$ au^{-1} and a similar full-width half-maximum. We have also compared the q-distribution of new comets with $a > 10^3$ au in the region $q < 5$ au (Fig. 7). In this case the number versus q increases approximately as $N(q) \propto (1 + 0.59q)$ (or $N(q) \propto (1 + 0.24q)$ in the region $0 < q < 3$ au), qualitatively similar to the $N(q) \propto (1 + 0.13q)$ found by Wiegert & Tremaine (their fig. 10b). The modest increase in the relative number of new comets versus q shown in Fig. 7 arises from the dynamical evolution of objects that have experienced previous passages through the planetary system at large perihelion distance ($5 < q < 35$ au), as discussed by Wiegert & Tremaine (cf. Fig. 4). In contrast, Fig. 6 shows an almost uniform q-distribution in the region $q < 5$ au, because the vast majority of objects that can reach $q < 5$ au without first passing through the outer planetary region are objects with $a \gtrsim 3 \times 10^4$ au and these tend to have an isotropic velocity distribution in the Oort cloud. For the same reason, the

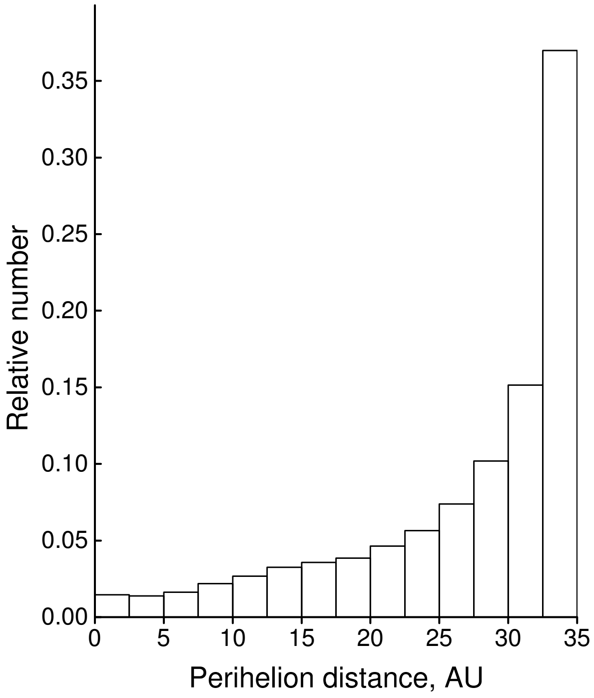

Figure 6. The relative number of dynamically new objects with $a > 1000$ au for different perihelion distances in the region $q < 35$ au.

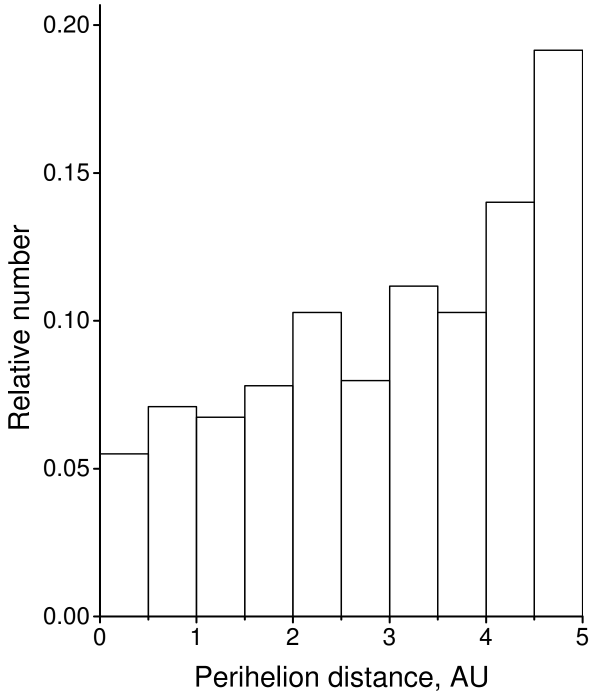

Figure 7. The relative number of dynamically new objects with $a > 1000$ au for different perihelion distances in the region $q < 5$ au.

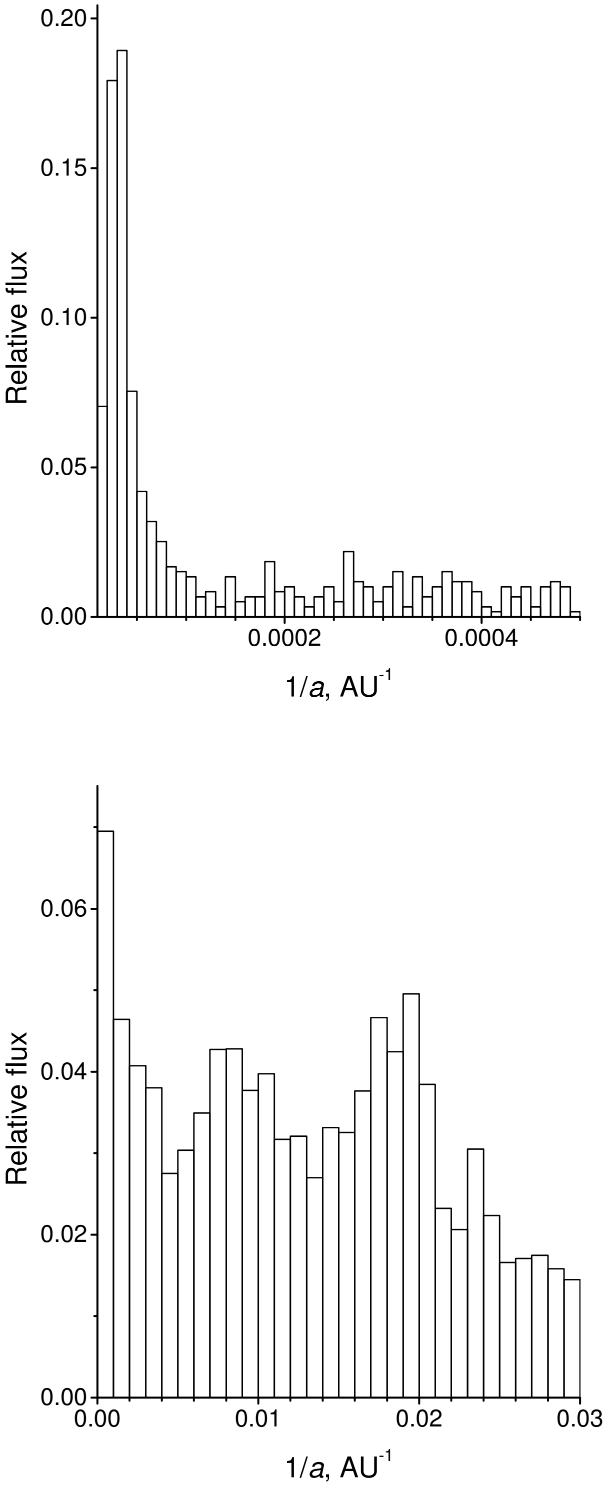

Figure 8. The relative flux, i.e. the relative number of perihelion passages, of objects in the region $2.5 < q < 5$ au as a function of $1/a$, shown on two different scales of $1/a$.

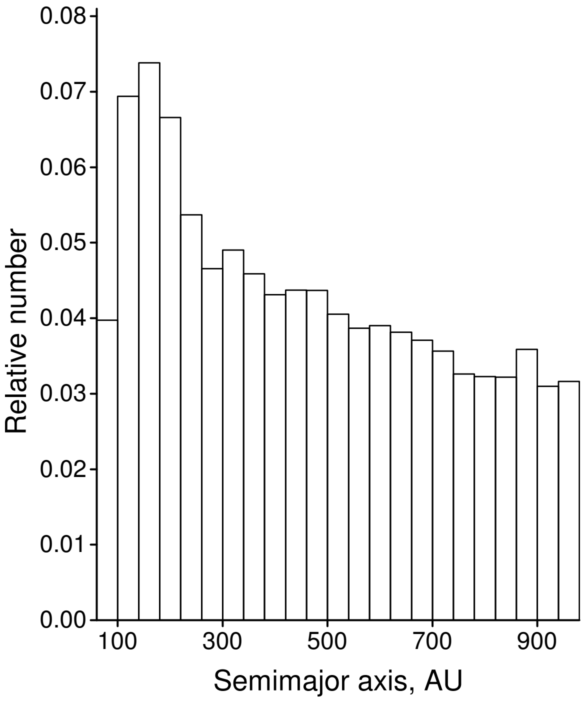

Figure 9. The a-distribution of Oort Scattered Disc (OSD) objects, i.e. high-eccentricity TNOs with $q > 30$ au and $60 < a < 1000$ au coming from the Oort cloud, during the last 1 Gyr of our integrations.

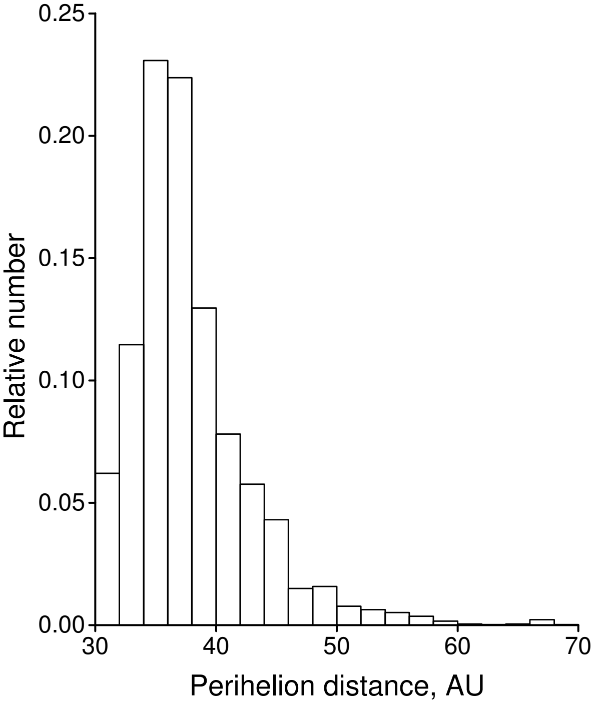

Figure 10. The q-distribution of Oort Scattered Disc (OSD) objects, i.e. high-eccentricity TNOs with $q > 30$ au and $60 < a < 1000$ au coming from the Oort cloud, during the last 1 Gyr of our integrations.

overall q-distribution shown in Fig. 6 flattens considerably if the lower limit on a is increased to 3×10^4 au or more, in accordance with expectations based on a loss-cone filling argument.

Finally we present (Fig. 8) the relative flux of comets versus $1/a$, on two different scales, which may be compared with Wiegert & Tremaine's fig. 16. Fig. 8 includes data from all perihelion passages (not restricted to dynamically new objects) within $2.5 < q < 5$ au, in the interval 50–200 Myr of the additional integrations. Although this is not exactly the q range they use (their fig. 16 is for $q < 3$ au) as the scope of our study differs from theirs, there is nevertheless a good level of quantitative agreement in the distributions, supporting the reliability of both studies. The comparison of this distribution with the observed $1/a$-distribution is of course complicated by the fading problem.

4 CONNECTION WITH TRANS-NEPTUNIAN OBJECTS AND CENTAURS

Our computations show that many objects in the Oort cloud can reach orbits typical of high-eccentricity TNOs. However, whereas the structure of the Oort cloud is determined mainly by the long-term action of planetary, stellar and Galactic perturbations, the influence of initial conditions remains strong for the trans-Neptunian region. For example, many objects in the 'classical' EKB evidently persist in stable orbits for the age of the Solar system. Similarly, many high-eccentricity TNOs can survive in this region for the age of the Solar system, especially those with initial $q > 36$ au which were not considered in our model. The most remarkable example is the object (90377) Sedna, which evidently provides information on some of the early processes associated with the formation of the Solar system (Morbidelli & Levison 2004; Brasser, Duncan & Levison 2006).

For these reasons it is difficult to draw conclusions on the general distribution of TNOs from our simulations, and so we consider only those objects that come to the TNO region from the Oort cloud. This subset of TNOs is fully modelled by our integrations.

4.1 TNOs from the Oort cloud, and the NNHE objects

We define the 'Oort Scattered Disc' (OSD) as the set of high-eccentricity TNOs with $q > 30$ au and $60 < a < 1000$ au that have at some time been in the Oort cloud ($a > 10^3$ au) and survived to the present day. After 4.5 Gyr, there were 328 such particles (Table 1). For ease of comparison with earlier studies on the capture of TNOs to the inner planetary region (Emel'yanenko, Asher & Bailey 2004) we also distinguish (cf. Table 1) the near-Neptune high-eccentricity (NNHE) region, defined by $28 < q < 35.5$ au and $60 < a < 1000$ au, an area of phase space that is quantitatively similar (but not identical) to the Scattered Disk introduced by Duncan & Levison (1997). Scaling these numbers by the total number of clones surviving in the Oort cloud to the present day and

Table 3. The numbers of OSD objects (N_S), with $q > 30$ au and $60 < a < 1000$ au; NNHE objects (N_N), with $28 < q < 35.5$ au and $60 < a < 1000$ au; and Centaurs (N_C), with $5 < q < 28$ au and $a < 1000$ au, coming from the Oort cloud at the present epoch for different models defined by initial distributions of perihelion distance in the early Solar system.

Initial q-distribution	$N_S/N_{\rm OC}$	N_C/N_N	N_S	N_N	N_C
Constant for $25 < q < 36$ au and zero otherwise	0.061	0.118	3.2×10^{10}	1.0×10^{10}	1.1×10^{9}
Constant for $5 < q < 36$ au and zero otherwise	0.037	0.129	1.8×10^{10}	0.5×10^{10}	0.7×10^{9}
$\sim 1/q$ for $5 < q < 36$ au and zero otherwise	0.028	0.138	1.3×10^{10}	0.4×10^{10}	0.6×10^{9}
$\sim 1/q^2$ for $5 < q < 36$ au and zero otherwise	0.019	0.153	0.9×10^{10}	0.3×10^{10}	0.4×10^{9}

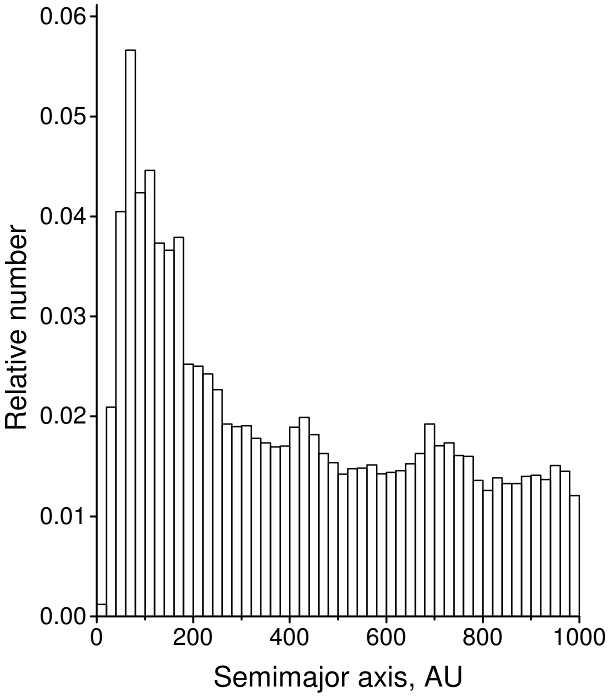

Figure 11. The a-distribution of objects from the Oort cloud coming to the Centaur region ($5 < q < 28$ au, $a < 1000$ au) during the last 1 Gyr of our integrations.

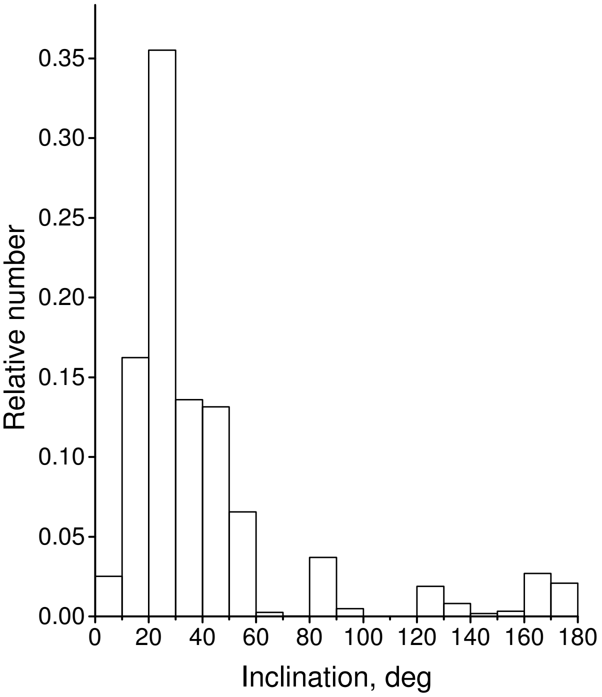

Figure 12. The i-distribution of objects from the Oort cloud coming to the Centaur region ($5 < q < 28$ au, $a < 1000$ au) during the last 1 Gyr of our integrations.

multiplying by the total number (Table 2) of comets in the Oort cloud gives the number of such objects surviving to the present day (Table 3).

Figs 9 and 10 show the frequency distributions of a and q for the OSD objects based on the last 1 Gyr of our main integration results. The a-distribution has a maximum somewhere between 100 and 200 au, and decreases gradually beyond this. This shows that there are many objects with large a which have not yet been discovered owing to severe observational selection effects. The q-distribution is consistent with the observed one (Emel'yanenko 2002b; Morbidelli, Emel'yanenko & Levison 2004). Moreover, Fig. 10 (see also Fig. 1) shows that an appreciable number of TNOs coming from the Oort cloud have $q > 40$ au; indeed, the maximum value of q is 67 au for objects in the region $a < 100$ au, and exceeds 100 au for TNOs in the region $a < 1000$ au. It is important to note, as shown in Fig. 1, that our evolved Oort cloud model produces objects with orbital elements similar to those of the exceptional object (148209) 2000 CR_{105}.

Table 3 gives estimates of the number of objects N_S in the OSD region and the number N_N in the NNHE region, for the different models described in Section 3. We see that N_S is approximately three times larger than N_N, i.e. the majority of objects that constitute N_S have $q \gtrsim 36$ au (cf. Fig. 10). As the values are derived from the numbers of particles populating the various regions relative to the number of Oort cloud particles, the comments in Section 3 about the possible slight variation of these estimates (e.g. a factor ∼1.4 smaller, owing to the uncertainties associated with close stellar encounters), apply here as well.

It is noteworthy that the estimate of N_N is consistent with the number obtained by considering these NNHE objects as a source of JFCs (Emel'yanenko et al. 2004). Furthermore, the orbital distribution of these former Oort cloud objects is in extremely good agreement with observational data. Thus our results show that the majority of observed near-Neptune high-eccentricity objects may come from the Oort cloud.

Table 4. The injection rate per year from the Oort cloud to Jupiter-family comets with $q < 2.5$ au for different models.

Model	ν_{JF}
$f(q) = \text{const},\ 25 < q < 36$ au	0.31
$f(q) = \text{const},\ 5 < q < 36$ au	0.16
$f(q) \sim 1/q,\quad 5 < q < 36$ au	0.11
$f(q) \sim 1/q^2,\quad 5 < q < 36$ au	0.07

4.2 Centaurs

Oort cloud objects can also penetrate the Centaur region, which we define by $5 < q < 28$ au, $a < 1000$ au. It was shown earlier (Emel'yanenko, Asher & Bailey 2005; Emel'yanenko 2005) that both direct injection from the inner Oort cloud due to Galactic and stellar perturbations and capture by dynamical transfer through the NNHE region are substantial. In the current paper we derive from our integration results the ratio of the number N_C of Centaurs originating from the Oort cloud to that of NNHE objects (Table 3). The calculated values of 0.12–0.15 are very close to the value of 0.13 corresponding to the debiased observed distribution found by Emel'yanenko et al. (2005). Moreover, that paper showed that Centaurs with $a < 60$ au, which are those usually discovered, number only about 10 per cent of the total in the Centaur region once observational biases are accounted for, and that the intrinsically more numerous Centaurs with $a > 60$ au are inconsistent with a proximate NNHE source. Emel'yanenko et al.'s (2005) suggestion, that the Oort cloud by contrast produces mainly Centaurs with $a > 60$ au, is confirmed by our calculations (Fig. 11), and the i-distribution (Fig. 12) explains why most discovered Centaurs have prograde orbits.

5 CONNECTION WITH SHORT-PERIOD COMETS

In order to estimate better the contribution of the Oort cloud to short-period objects, which we take to mean objects with periods $P < 200$ yr, we analysed our additional integrations (Section 3) of 20 cloned orbits for each object surviving after 4.5 Gyr. These calculations show that both Jupiter-family and Halley-type comets can originate from the Oort cloud. Over 150 Myr we registered nine objects with Tisserand parameters, T, with respect to Jupiter $T > 2$, and three objects with $T < 2$ on short-period orbits when their perihelia first dropped below 2.5 au. All the nine JFCs and two of the three HTCs originated from initial orbits with q in the range 25–36 au, and the other HTC originated in the 10–25 au region.

5.1 Jupiter-family comets

The ratio of the new-comet flux with $q < 5$ au to the JFC injection rate is 560/9 (cf. Section 3) for the model in which the initial q-distribution is uniform in the range 5–36 au, and so the adopted observed new-comet flux of 10 comets per year (Section 3) yields a JFC injection rate ν_{JF} to Jupiter-family orbits (defined as $T > 2$ and $q < 2.5$ au) of 0.16 per year (Table 4). The Table also shows values of ν_{JF} for the other models introduced in Table 2.

These results show that the Oort cloud provides a significant number of JFCs. To demonstrate this, let us consider JFCs with $q < 1.5$ au. The injection rate to $q < 1.5$ au is approximately half the rate to $q < 2.5$ au (Emel'yanenko et al. 2004). The values in Table 4 therefore imply that a mean physical lifetime of only 580–2600 yr for JFCs in the region $q < 1.5$ au is sufficient to explain a steady-state number of about 90 comets in this region (Fernández et al. 1999).

On the other hand, it is evident that JFCs can come from different sources in the outer Solar system, e.g. from the primordial scattered disc of Duncan & Levison (1997) or from resonant TNOs (Morbidelli 1997). Nevertheless, our present work demonstrates a potentially dominant contribution from objects that have visited the Oort cloud, consistent with results from the Deep Impact mission (Mumma et al. 2005; Sugita et al. 2005), although the exact fraction of each source population among JFCs remains uncertain. In this situation, estimates of the physical lifetime of JFCs based on investigations of a single source region (e.g. Levison & Duncan 1997) would have to be reconsidered.

5.2 Halley-type comets

As to Halley-type comets, our estimates of the injection rate ν_{HT} are very uncertain because we registered only three objects captured to $q < 2.5$ au with $T < 2$ in our calculations (recall that we stopped our calculations at $q < 2.5$ au and thus did not study the usual diffusion process of capture from the near-parabolic flux with small perihelion distances, investigated for example by Emel'yanenko & Bailey 1998 and Levison et al. 2001). The capture of one Halley-type object happened after a direct injection from the outer Oort cloud to a near-parabolic orbit with $q = 3.3$ au, but the other two reached Halley-type orbits through gradual dynamical evolution from the outer Solar system through the planetary system, the evolution including time spent as a Centaur (Figure 13; cf. Emel'yanenko 2005).

This possibility, namely that HTCs can also originate from high-eccentricity orbits with large q, is very important. Whereas during evolution at small q, comets can disintegrate before reaching short-period Halley-type orbits (Levison et al. 2002), they can survive longer at large distances where they first evolve to short periods, only afterwards changing q gradually to small values. In such cases the physical and dynamical evolution from the outer Solar system to HTCs is reminiscent of that for JFCs.

If we take only objects injected into the planetary region with $q > 5$ au, our preliminary estimates show that the ratio ν_{HT}/ν_{JF} is more consistent with observational data on the numbers of JFCs and HTCs than the case of direct capture at small q. Therefore it is quite possible that the long-standing problem of the number ratio of Halley-type to Jupiter-family comets is connected with the mechanism of their evolution from the outer Solar system to short-period orbits and not just with a physical difference between these comets (Bailey 2002).

6 CONCLUSIONS

We have developed a realistic model of the present day Oort cloud by accounting for planetary, stellar and Galactic per-

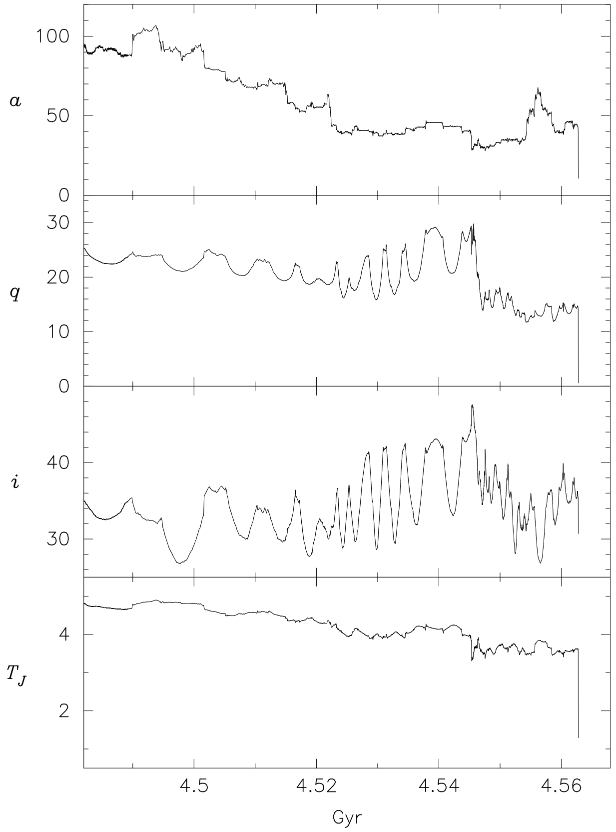

Figure 13. The last c.80 Myr evolution of a clone that became a Halley-type comet by gradual dynamical transfer from the outer Solar system. Plotted are a, q, i and the Tisserand parameter with respect to Jupiter. In the orbital classification scheme developed by Horner et al. (2003) this clone evolved from an $\mathrm{NT}_{\mathrm{II}}$ object under the control of Neptune to a $\mathrm{UT}_{\mathrm{III}}$ object under the control of Uranus. It then entered a $\mathrm{UE}_{\mathrm{III}}$ and a $\mathrm{UE}_{\mathrm{II}}$ state, before a close planetary encounter finally injected it into the SP_{I} Halley-type comet region.

turbations in numerical integrations covering 4.5 Gyr. The model is used to provide details of the various cometary populations injected by the Oort cloud into different dynamical regions of the planetary system. Although it has been shown elsewhere that the Edgeworth-Kuiper belt is a source of Centaurs and Jupiter-family comets, here we have demonstrated the fundamental role of the Oort cloud in determining the flux of cometary bodies through the planetary system. This includes an Oort cloud origin for near-Neptune high-eccentricity objects, Centaurs and both Jupiter-family and Halley-type short-period comets. Our estimates of the numbers of such bodies are somewhat uncertain, but we emphasize that their numbers and orbital characteristics are consistent with observations, so unifying an extensive body of observational data and indicating that a substantial fraction of all known cometary bodies may have a proximate source in the Oort cloud.

ACKNOWLEDGMENTS

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