Stellar Magnetism
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OBSERVING AND MODELLING STELLAR MAGNETIC FIELDS. 2. MODELS

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Abstract. In this chapter, we examine the question of how spectropolarimetric observations of magnetic stars may be modelled, and how modelling techniques may be used to extract much detailed information about the stellar magnetic field and other characteristics of the magnetized stellar atmosphere.

1 Introduction

In the preceding chapter, we discussed how the energy levels of atoms are split by a magnetic field, and how this leads in turn to splitting of spectral lines in to multiple components, an effect called the Zeeman or Paschen-Back effect depending on the field strength. We saw that the polarisation characteristics of these components provide valuable tools for detection and measurement of stellar magnetic fields, and that polarisation measurements allow one to obtain, in a straight-forward way, an estimate of the mean line-of-sight field component averaged over the visible hemisphere (the mean longitudinal field $\langle B_z \rangle$). We also saw that, if the field is large enough and $v_{\rm e} \sin i$ small enough, observation of splitting of spectral lines into discrete components makes it possible to estimate the mean field modulus $\langle B \rangle$, again averaged over the visible hemisphere. Finally, we looked at some simple multipole models of stellar field structure that can reproduce simple data-sets such as observed values of $\langle B_z \rangle$, or of $\langle B_z \rangle$ and $\langle B \rangle$, over a stellar rotation, and how these models lead to the fundamental conceptual model known as the "oblique rotator".

Determination of mean field values "directly" from the spectra (in the same way as radial velocities and projected rotation velocities are derived), and the modelling of such integral or moment data with simple geometric field structures, provide a useful and valuable source of information about global characteristics of the stellar field. However, such models are based on data-sets that are often

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no more complex than one or two sine waves of $\langle B_z \rangle$ and/or $\langle B \rangle$ variation with rotation. These models cannot provide detailed information about the structure of the field, or about local relationships between the field and abundance variations of specific elements over the stellar surface.

We now consider a much more complex and powerful class of models. The basic idea is to model in detail the shape and polarisation of one or many spectral lines by computing the expected emergent spectrum of a star with a hypothesized field geometry (and chemical abundance distribution of the elements whose spectral lines are studied). The detailed line properties are then compared with the spectrum observed at numerous phases in the rotation cycle, and the field structure and abundance model are iterated to obtain good agreement between the computed model and the data. Because this type of modelling allows a much more detailed comparison between the hypothesized field and abundance maps and the observations, a much more detailed map of the star can be derived than by using simple integral field quantities such as $\langle B_z \rangle$. Furthermore, discrepancies that remain after the best possible convergence has been obtained provide valuable hints about geometrical or physical effects omitted from the calculated model.

Most of the work in using this modelling technique is in the computation of the emergent spectrum of the magnetized star, and so we need to give some attention to the problems of "spectrum synthesis" of magnetic stars. Two essential aspects of this problem arise, that are usually treated with different computational tools and approximations.

- We must model the physical conditions in the layers of the star from which radiation emerges – we must compute a suitable model atmosphere. That is, we need to determine the variation of temperature $T(z)$, pressure $p(z)$, density $\rho(z)$, etc, with altitude in the atmosphere, possibly as a function of position over the stellar surface.

- Once the model atmosphere is available, we need to compute the detailed spectrum of radiation emerging from it in some spectral window for which we have data. This means that we need to determine the "specific intensity" $I_\nu(\theta)$ of radiation emerging from many sample points on the model star, at an angle θ with respect to the local vertical, in the direction towards the observer.

The atmosphere model is computed assuming that the radiating region is geometrically thin (so that it can be treated as flat), that it is locally chemically homogeneous horizontally (and often vertically homogeneous as well), and that it is in hydrostatic and thermal equilibrium. The model atmosphere is often computed using simplifying assumptions that are dropped in the second step; for example, all the effects of the magnetic field may be ignored in this step.

In the second step, the effects of the magnetic field on spectral lines, and the detailed variations of chemical abundance, are included into the model whose emergent spectrum is computed. This model is calculated essentially by solving the "equation of radiative transfer" at many points on the model star's surface,

and adding up the emergent radiation rays in a suitable way. Often it is only this step that is iterated to bring the model of the magnetic star into agreement with the available data.

Because the physics and mathematics involved in spectrum synthesis is often unfamiliar, or at least somewhat difficult to understand, we will review the basic ideas of such computations. The emphasis here will be on providing ways of understanding physically how synthetic spectra are obtained and what their essential information content is, rather than exploring detailed computation methods or providing proofs of various important results.

2 The equation of radiative transfer: a simple introduction

For many people the equation of radiative transfer, especially the form including the effects of a magnetic field, is somewhat obscure. However, if we look at the basic ideas involved, much of this obscurity may be cleared away, or at least transferred over into the category of complicated detailed methods for solving differential equations, etc. A good elementary discussion of stellar atmosphere physics is found in the textbook of Novotny (1973).

It is convenient to start this discussion by looking at the form and nature of the equation of transfer in a particularly simple situation, that of transfer of *unpolarised* radiation in a purely absorbing and thermally radiating gas, in order to build some physical understanding. After looking at this case, we extend the discussion to polarised radiative transfer.

The fundamental idea is that we will describe the radiation field at any point inside the stellar atmosphere, and in the space outside the star once this radiation has emerged, by the natural physical quantity, the specific intensity $I_\nu(\theta)$. If you think about the units of this quantity for a moment (erg s^{-1} cm^{-2} Hz^{-1} $sterad^{-1}$) you will realise that this is the completely natural quantity to describe the energy flowing in the radiation field in enough detail to allow the computation of a spectrum. The only unexpected point may be the fact that we usually consider the radiation per unit frequency (i.e. per unit photon energy) rather than per unit wavelength, but transformation from one to the other is trivial (try it).

Now consider radiation flowing in all directions inside a hot gas which is emitting and absorbing (but not scattering) this radiation. We describe the absorption by an absorption coefficient, a function of frequency, that describes the extinction per unit length ds of path travelled by a ray of radiation through the gas: $dI_\nu = -\kappa_\nu I_\nu ds$. Clearly, κ_ν has the dimensions of fractional loss per cm (cm^{-1}), or equivalently of cross section cm^2 per cubic cm of gas (cm^2 cm^{-3}). Similarly, we describe the thermal emission at frequency ν by the emissivity per unit volume, j_ν, in ergs s^{-1} cm^{-3} Hz^{-1} $sterad^{-1}$. We then write the radiation added to the ray we are following in ds of path as $dI_\nu(\theta) = j_\nu ds$. The net change in the specific intensity (gains minus losses) after travelling a distance ds through the gas is

$$dI_\nu = (-\kappa_\nu I_\nu + j_\nu)ds. \tag{2.1}$$

To feel more comfortable with this equation it is really important to think about the physical units used for each quantity, and why this simplest equation of transfer is written in the form above.

Now of course to compute the spectrum of light emerging from the atmosphere of the whole visible hemisphere of the star, we must consider radiation travelling in all directions relative to the atmosphere's local vertical (z) axis, even though with our assumptions the flow of radiation in any direction and at any wavelength is independent of – decoupled from – the flow in any other direction or at any other frequency. Distance ds along any ray is related to dz and the angle θ between the ray and the local vertical by $dz = \cos\theta ds = \mu ds$. Now we define the very useful quantity of (monochromatic) optical depth τ_ν by $d\tau_\nu = -\kappa_\nu dz$. Notice that optical depth is a natural measure of how far we can see into the stellar atmosphere, or from what depth radiation can emerge unchanged for us to see. Radiation flowing through a distance corresponding to one unit of optical depth has a large probability of being absorbed, an so we will see little of the radiation that is flowing outward from greater optical depth than this. Most of the radiation that actually emerges from a stellar atmosphere comes from regions above the level of $\tau_\nu \sim 1$ (measured from the top of the atmosphere). The equation of radiative transfer (ERT) for a ray of frequency ν, moving at angle θ to the local vertical, is then conveniently written as

$$\mu \frac{dI_\nu}{d\tau_\nu}(\tau_\nu, \theta) = I_\nu - \frac{j_\nu}{\kappa_\nu} = I_\nu - S_\nu. \tag{2.2}$$

Several points about this form of the equation should be kept in mind.

- This form of the ERT ignores scattering processes, which directly couple rays moving in various directions and greatly complicate the problem.

- In this approximation, each ray has its own ERT, independent of all the other ERT's describing other rays. Coupling of radiation travelling in one direction to that going in another direction only happens by thermal absorption and re-emission by the gas.

- If we start any particular ray deep in the atmosphere, well below the $\tau_\nu = 1$ ("optical depth unity") level and follow it outward to the surface, Eq. (2.2) predicts the local emerging intensity in direction θ, $I_\nu(0, \theta)$.

A great further simplification can be obtained by assuming that the thermal emission is in "local thermodynamic equilibrium" and may be described by the Planck function $B_\nu(T)$ (which has the same dimensions – surprise – as the specific intensity). In this case $S_\nu = B_\nu(T)$ has a form which is *given* once the local temperature $T(z)$ of the model atmosphere is known.

Now if S_ν is a specified function of optical depth (this would be the task of the model atmosphere), Eq 2.2 is just a linear, first-order ordinary differential equation with a driving term. It can be solved by a number of methods. For example, we

could multiply both sides of the equation by the integrating factor $\exp(-\tau_\nu/\mu)$, resulting in an expression that may be integrated directly to give

$$I_\nu(\tau_\nu) = e^{\tau_\nu/\mu} \int_{\tau_\nu}^{\infty} S_\nu(\tau'_\nu) e^{-\tau'_\nu/\mu} d\tau'_\nu/\mu \tag{2.3}$$

If we evaluate this at $\tau_\nu = 0$ (take the lower limit of integration to be 0) we have the specific intensity emerging from the stellar surface.

Now let's see what this means in a concrete example. We will try the simplest functional form for the source function which simply increases inwards: take $S_\nu(\tau_c) = S_0(1+\beta\tau_c)$ where τ_c is measured at some convenient continuum frequency in the spectral window of interest. In this case we can evaluate the integral (try it!):

$$I_c(0) = S_0(1 + \beta\mu) = S_c(\tau_c = \mu). \tag{2.4}$$

That is, given the source function (i.e. we have a model atmosphere which we are studying), this expression gives us the emerging specific intensity of radiation at different inclinations μ to the vertical.

Now what happens in a spectral line? Suppose that across the line we know $\kappa_\nu = \kappa_c + \kappa_{\rm line} = (1+\eta_\nu)\kappa_c$. The function of frequency η_ν describes the line profile. Suppose (again for simplicity and ease of calculation) that η_ν is independent of depth in the atmosphere. Then $\tau_\nu = (1 + \eta_\nu)\tau_c$, and consequently we may write the source function as a function optical depth in the line as $S_\nu(\tau_\nu) = S_0[1 + \beta\mu\tau_\nu/(1 + \eta_\nu)]$. Then we have the same problem as before, resulting in a very similar expression for the emergent specific intensity through the line:

$$I_\nu(0) = S_0 \left(1 + \frac{\beta\mu}{1 + \eta_\nu}\right). \tag{2.5}$$

Clearly the line opacity profile (which is usually a Voigt profile) has a large value near line centre and drops off into the wings, and correspondingly the emergent specific intensity is large in the continuum but decreases through the line.

The point of this whole discussion is that in this case the ERT is a *solvable first order, linear differential equation*. Of course, there are a lot of harder cases, but as long as we stay away from considering scattering, the ERT remains relatively tractable, although usually numerical methods of solution are required.

3 Polarised radiative transfer

Next we turn to the question of how the equation of transfer has to be modified to deal with polarised light. We start by looking at a very useful description of polarisation, by means of the Stokes vector, and then survey some results from polarised radiative transfer. For anyone who wants to get deeper into this very interesting subject, two important recent textbooks are "Solar Magnetic Fields: Polarized Radiation Diagnostics" by Stenflo (1994), and "Introduction to Spectropolarimetry" by del Toro Iniesta (2003). Note that although these books nominally focus on solar polarimetry, they both present extensive introductions to polarised radiative transfer that are much more general than the solar case.

3.1 The Stokes vector

To describe light mathematically, we use the Stokes vector $[I, Q, U, V]$. In this vector

- I is the total (specific) intensity of light in the beam.
- Q and U describe the linear polarisation of the beam. We can define them by measurements. Using a perfect linear polariser, which transmits one linear polarisation with 100% efficiency, and completely blocks the orthogonal polarisation (for example, an anti-reflection coated Nicol or Glan-Thompson prism; commercial plastic sheet polariser is conceptually similar but imperfect), measure the intensity of the beam with the transmission axis of the polariser set to 0°, 45°, 90°, and 135°. Then $Q = I_0 - I_{90}$ and $U = I_{45} - I_{135}$. Thus, Q measures the extent to which the light polarised at at 0° is more or less intense than that polarised at 90°. The larger this difference, the more polarised the beam, to the limit where all of I is in one of the two analysed beams.
- Now measure the intensity of the beam through two perfect analysers one of which completely transmits left circularly polarised light, but completely blocks right circularly polarised light, and the other of which does the opposite: $V = I_{\rm right} - I_{\rm left}$.

These four quantities adequately describe the polarisation state of the beam for our purposes. Notice that $[I, Q, U, V]$ are in general functions of both frequency and direction, just as the normal specific intensity I is (for further details, see Shurcliff 1966; Born & Wolf 1965).

Sometimes the polarisation components Q, U, V are normalised with respect to I: Q –> Q/I, etc; in this case the polarisation components are expressed as fractional or percentage polarisation.

3.2 Equations of radiative transfer for polarised radiation

It will not be surprising to learn that, just as in normal radiative transfer, we require as inputs the ratios of total (line plus continuum) to continuum opacity to completely specify the physical inputs for a spectrum calculations. We define the quantities $\eta_{\rm p}$, $\eta_{\rm l}$ and $\eta_{\rm r}$ as the ratios of total (Voigt line profiles plus continuum) opacity to continuum opacity alone, for the π, left-polarised σ, and right-polarised σ components of the Zeeman pattern of a line. Further, define ψ as the angle between the direction of the ray we are interested in and the magnetic field vector, and choose the zero of the angle measuring Q as the plane of the ray and the magnetic field. Then define

$$\begin{aligned} \eta_I &= 0.5\eta_{\rm p}\sin^2\psi + 0.25(\eta_{\rm l}+\eta_{\rm r})(1+cos^2\psi) \\ \eta_Q &= 0.5\eta_{\rm p} - 0.25(\eta_{\rm l}+\eta_{\rm r})\sin^2\psi \\ \eta_V &= (\eta_{\rm r}-\eta_{\rm l})\cos\psi \end{aligned} \tag{3.1}$$

Notice that η_Q and η_V are *differences* of Zeeman opacities, analogous to the definitions of Q and V themselves. These terms act to introduce linear and circular polarisation into the ray we are following.

Similar expressions (involving terms we label as ρs) are required to describe retardance ("anomalous dispersion") of the ray.

The equation of radiative transfer for polarised light becomes four equations, one for each Stokes component. The credit for realising that this is the optimal way to express the equation of transfer for polarised light belongs to Unno (1956); his paper is still well worth reading. These four equations look a lot like the ERT for unpolarised light, Eq 2.2:

$$\begin{aligned}
\mu\frac{dI}{d\tau_\nu} &= \eta_I(I - B_\nu) + \eta_Q Q + \eta_V V \\
\mu\frac{dQ}{d\tau_\nu} &= \eta_Q(I - B_\nu) + \eta_I Q - \rho_R U \\
\mu\frac{dU}{d\tau_\nu} &= \rho_R Q + \eta_I U - \rho_W V \\
\mu\frac{dV}{d\tau_\nu} &= \eta_V(I - B_\nu) + \rho_W U + \eta_I V. \qquad (3.2)
\end{aligned}$$

These equations look more formidable than the normal equation of transfer, but they are structurally almost exactly the same. This is a system of four coupled ordinary, first order, linear differential equations, and can be solved by essentially the same methods that are used for a single equation. Notice that, again, this formulation ignores any effects of scattering (or treats it as absorption), and the source function is assumed to be the Planck function (i.e this is LTE). Polarisation is introduced into the ray of interest essentially by the Zeeman components of the lines, as one would expect. The notation here follows that of Martin & Wickramasinghe (1979), which also has a good discussion of solution methods.

For a given stellar atmosphere structure, we start the solution of these four equations well below the level of optical depth 1 where we assume that the ray of interest is unpolarised, and follow the ray outward to the top of the atmosphere, where it becomes the local emergent specific intensity, now expressed in terms of a Stokes vector. For a complete solution of the polarised radiation from the star in some region of its spectrum, we must follow a large number of such rays out from beneath the atmosphere, for a large enough number of frequencies to adequately sample line profiles.

We cannot accurately model the intensity *or* polarisation of the light from a magnetic star without solving this whole system of equations. Even the intensity spectrum is substantially different from what is computed from a single unpolarised ERT, whether one treats the spectral lines as Zeeman split or not

As in the case of the single ERT, for the simplest case of a linear source function, there is an analytical solution. The simplest form of this solution (ignoring anomalous dispersion effects) was derived by Unno (1956). The main results are

reproduced here, and are worth experimenting with to build intuition.

$$\begin{aligned} \frac{I_{\rm c}(0,\theta)-I(0,\theta)}{I_{\rm c}(0,\theta)} &= \frac{\beta\cos\theta}{1+\beta\cos\theta}\left[1-\frac{1+\eta_I}{(1+\eta_I)^2-\eta_Q^2-\eta_V^2}\right] \\ \frac{V(0,\theta)}{I_{\rm c}(0,\theta)} &= \frac{-\beta\cos\theta}{1+\beta\cos\theta}\left[\frac{\eta_V}{(1+\eta_I)^2-\eta_Q^2-\eta_V^2}\right]. \end{aligned} \tag{3.3}$$

4 Model atmospheres

One of the important components of the computation of a stellar spectrum, whether the star is magnetic or not, is the model stellar atmosphere used as a basis for the spectrum computation. In the kind of LTE modelling we are discussing, this model provides the essential function $T(z)$ and $T(\tau_\nu)$, and hence $B_\nu(\tau_\nu)$, which is the externally specified driving term in all the forms of the ERT discussed here. This model atmosphere is almost always computed in a separate suite of programmes from those used for the calculation of the observed spectrum.

The model atmosphere is computed, in principle, as follows. First a convenient frequency ν_0 is chosen as a reference frequency, and a form of the temperature structure $T(\tau_0)$ is assumed. Then using the equation of hydrostatic equilibrium, the structure of the atmosphere is calculated inward from the top, including a calculation of τ_0 (which of course requires evaluating the opacity at ν_0 as one goes inward) so that temperatures can be assigned to successively deeper grid levels. In this computation, local thermodynamic equilibrium is assumed.

When this step has been completed, the programme next computes the opacity at all frequencies (with some reasonable spacing) at which there is significant radiative flux. Then the equation(s) of transfer is (are) integrated outwards along many rays, and the integrated flux is calculated at each atmosphere level in the computation. The objective is to have the integrated flux (or luminosity) constant from bottom to top of the atmosphere (to some precision) so that there is no net accumulation or loss of energy at any level, but of course since the right $T(\tau_0)$ was not available initially, this is not achieved. The programme next computes a correction to the initial $T(\tau_0)$ at each level, designed to reduce the net accumulation or loss of energy at each level. (For example, naively one might try to raise the temperature at each level where energy accumulation occurs, and lower it where net loss occurs. In fact the correction procedures are considerably more refined than this.)

The computation of the atmospheric structure is repeated, and checked again for flux constancy. The temperature correction process is applied again, and the cycle is repeated, usually several times, until the radiative flux flowing out through the atmosphere is found to have the same value at all levels.

In the part of the programme concerned with the computation of the outward flux, the opacity must be evaluated at many wavelengths so that the flux is computed with sufficient accuracy. In modern programmes, this normally requires

inclusion of many (typically $\sim 10^6$) spectral lines, as the stronger lines in the spectrum are found to significantly alter the integrated flux of radiation outwards. In the case of a magnetic star, one would like to include the effects of Zeeman splitting and polarised radiative transfer on the integrated net flux whose constancy is required, and there is no reason not to do this except for the fact that it makes the computation of the model atmosphere much slower than in the non-magnetic case.

Until recently, the small number of codes for model atmosphere computation that are available were all for unpolarised radiation. The most commonly available of these are ATLAS (http://Kurucz.harvard.edu; Kurucz 1979), which is really optimised for 1D atmospheres in stars between the effective temperature at which molecules become important (about 5500 K) and the temperature at which winds begin to distort the structure ($\sim$ 20000 K), and MARCS (see http://www.phys.uwosh.edu/mike/exercises/marcs/marcs.html), which is designed for lower effective temperatures, roughly between 4000 and 7000 K. There is also the Phoenix model atmosphere code, which covers a wide range of conditions, including departures from LTE (Hauschildt & Baron 1999).

For the computation of model atmospheres using polarised radiative transfer and Zeeman split lines, the logic is essentially the same as discussed above, but of course the abundance table and the magnetic field configuration must both be specified. A programme for the computation of such models has recently been developed by Khan & Shulyak (2007 and references therein). They have done a large number of numerical experiments to determine the importance of various aspects of computation of magnetic star atmospheres, and conclude that it seems to be an acceptable approximation in many cases to use unpolarised model atmospheres, but they find that the specific abundance table used, especially with the kinds of overabundances often found in magnetic Ap stars, can have quite significant effects on the model atmosphere.

5 Computing an emergent spectrum

Given a suitable model atmosphere ($T_{\rm eff}$, $\log g$, abundances), what do we have to do to compute the emergent polarised spectrum of a magnetic star (the *forward* computation)?

- Assume some magnetic field structure, and calculate the vector field at many (50+) grid points on the visible hemisphere.
- For each grid point, compute the detailed run of polarised opacities and retardances at all the relevant depths (60+), at closely spaced frequencies or wavelengths (0.01 Å is barely an adequate wavelength grid in visible). A window of 100 Å might be a useful size.
- Then compute the emergent spectrum along the ray towards observer at each surface grid point, by solving the four ERTs outward along the ray, for each wavelength in the window.

A lot of bookkeeping is required!

Several aspects of this problem require special considerations.

- One needs to have a suitable list of spectral lines, with gf values and Landé splitting factors. The usual source of such data is now the VALD database at http://ams.astro.univie.ac.at/vald/ which supplies both these data (and many others) from a variety of sources. (Because some of these sources are more reliable than others, it is worth while getting fairly familiar with the details of the VALD database.)

- Because one must compute the Voigt and Faraday-Voigt functions millions of times, an efficient algorithm is needed. Good algorithms, for example in the form of complex rational polynomials, are now available.

- Solving the equations of transfer numerically may be done with standard packages or methods (e.g. a Runge-Kutta solver), but again this has to be done so many times that efficiency is essential, and you want a technique that functions well on as sparse a depth grid as possible.

Descriptions of common codes discuss these (and many other) points. If you use any of the available codes, it is very important to familiarise yourself with the basic tactics of the code, and with its strengths and limitations.

Zeeman (Landstreet 1988) is a simple magnetic line synthesis code. I discuss it here as an example of how such a code can function.

- Zeeman contains simple parametrised magnetic field structures (colinear dipole, quadrupole, etc), that are imposed on the stellar atmosphere and determine the polarisation introduced by the Zeeman effect in spectral lines. The programme supports a simple model of abundance variation over the stellar surface, in which abundance tables are specified on rings colinear with field axis. (Some such flexibility is required for Ap stars with their varying abundance tables over the surface .)

- The programme reads in fundamental parameters of the star, assumed magnetic field parameters, the spectral window to compute, an appropriate model atmosphere for the problem, and much required atomic data such as as oscillator strengths, excitation energies, and Zeeman patterns. Many of these data are obtained from VALD.

- Zeeman then computes I, Q, U, and V spectra, including line splitting and polarised transfer, using the precomputed ATLAS atmosphere and a selection of line data appropriate to the spectral window chosen.

- The programme then compares the computed spectrum with an observed spectrum, and selects a best fit $v_e \sin i$ and radial velocity, by minimising the χ^2 of the fit of the computed spectrum to the observed one.

- If desired, Zeeman can iterate the fit to optimise field or abundance parameters for a best fit to lines of the element being optimised.

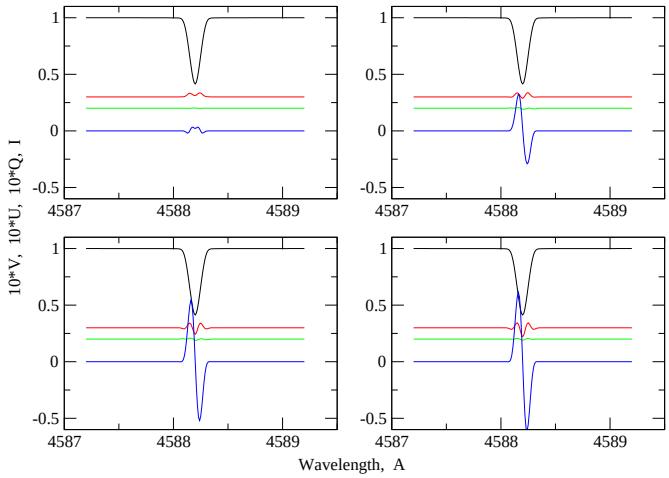

Fig. 1. Synthetic I, Q, U, and V line profiles of Cr II in an A0 main sequence star ($T_{\rm eff}$ = 9650 K, $\log g$ = 4.05 cgs units) that has a dipole magnetic field with a field strength of 1 kG at the poles and $v \sin i = 0$ km s^{-1}. The star is viewed at four inclinations from the magnetic pole: 90° (top left), 60° (top right), 30° (lower left), and 0° (lower right). In each panel the top profile is I, and below I are successively $10 \cdot Q$, $10 \cdot U$, and $10 \cdot V$.

An example of polarised line synthesis as computed with Zeeman is shown in Fig. 5. This figure illustrates the very simple case of a single isolated Cr II spectral line. The actual Zeeman pattern of the specific line is included in the computation, which is done assuming that Cr is about two times overabundant in the atmosphere relative to solar abundance.

Notice that the spectral line as viewed in I shows no trace of any magnetic signature (the field would have to be at least ten times larger for visible splitting to be seen even in this very slowly rotating star). However, quite a strong signature is visible in circular polarisation (the V profile at the bottom of each panel), which grows rapidly as the line of sight moves from the stellar equator (inclination of 90° from the pole) to the pole (inclination 0°).

Notice also that for this relatively modest (but fairly typical) field strength the linear polarisation signatures are always much smaller than the circular polarisation. Effectively, as discussed in the first chapter, the circular polarisation is a first order effect in field strength while the linear polarisation is second order. Clearly the field is far easier to detect in such a star using circular rather than linear polarisation.

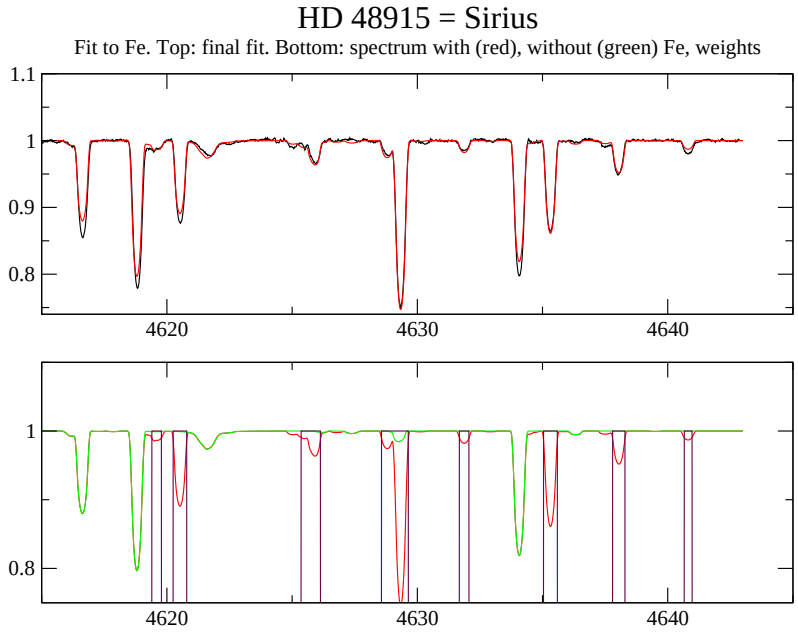

Fig. 2. An example of automated spectrum optimisation for the non-magnetic star Sirius A. In the upper panel the black spectrum is the observed spectrum, the red spectrum is a fit to Fe lines (but not yet to the three strong Cr lines). The lower panel compares a synthetic spectrum with and without Fe, to enable identification of useful Fe lines for the fit, and the vertical purple lines identify spectral regions in which the χ^2 of the fit is minimised.

Synthesis fits to non-magnetic stars may be very accurate. Such fits require good choices of $T_{\rm eff}$, $\log g$, abundances, radial velocity, $v_{\rm e} \sin i$, and the microturbulence parameter. $T_{\rm eff}$ and $\log g$ are often chosen from available Stromgren or Geneva photometry calibrations. Automated iterative fitting of most remaining parameters works well for such stars. An example of such fits is shown in Fig. 5.

An example of a fit to a magnetic star using Zeeman is provided by Fig. 5. The data available for this star are essentially the data foreseen when Zeeman was designed in the 1980's: short I spectra with reasonably uniform and dense phase coverage (here the spacing is about 0.1 cycle from one spectrum to the next); longitudinal field $\langle B_z \rangle$ measurements, and information about the mean field modulus $\langle B \rangle$ from line splitting in the spectrum. The model found for these

data fits all these data reasonably well, but as we will see later, is unlikely to provide a very accurate representation of the actual distribution of abundances and magnetic field vector over the stellar surface. Nevertheless, such a fit does provide useful information about the general strength of the field and about characteristic abundances (and their variations) of several elements, notably Si, Cr, and Fe.

As HD 215441 rotates, the line of sight moves from close to one magnetic pole (top and bottom spectra in Fig. 5) to nearly on the equator (middle spectra). This allows us to estimate the abundances of various elements roughly from pole to equator in one hemisphere. We find that all the elements visible in this segment of the spectra decrease in abundance from the pole to the equator, typically by a factor of ten or more (note that essentially all the spectral lines are weaker in the middle spectra than in the top and bottom spectra), but even at the equator Si, Ti, and Cr are at least ten times more abundant (relative to H) than in the Sun (see Landstreet et al. 1989 for more details, although this model is derived using a more recent version of Zeeman than is described there).

6 Recent observational advances

To advance beyond the relatively simple synthesis and modelling described above, we need more extensive data. A major advance during the past decade has been the development of facility instruments capable of high resolution spectropolarimetry in all four Stokes components. The instruments that have had the most impact are the MuSiCoS spectropolarimeter at Pic du Midi (Wade et al. 2000a), and its successors ESPaDOnS (at CFHT) and Narval (at Pic du Midi), all developed essentially by J.-F. Donati with the support of the team of engineers of l'Observatoire Midi-Pyrenées in Toulouse.

MuSiCoS provided spectra of resolving power $R = 35\,000$ in the spectral window 4600 – 6600 Å. Although this inexpensive instrument was not very efficient, data from it provoked several major break-throughs in various fields of stellar magnetism. ESPaDOnS and Narval, which are very similar to each other, have $R = 68\,000$, a spectral window running from 3700 to 10400 Å, and throughput increased by more than a factor of ten over MuSiCoS.

These instruments provide very respectable measurements of all four Stokes components in single spectral lines, but the the sensitivity of these instruments for detecting really weak signals has been greatly enhanced by a powerful technique for combining signals from many spectral lines coherently to greatly improve the resultant SNR (signal-to-noise ratio). This method, called Least Squares Deconvolution, or LSD, has made it quite easy to extract extremely precise field moments, especially $\langle B_z \rangle$, from spectropolarimetry (Donati et al 1997). LSD works essentially by rescaling a long list of spectral lines in a spectrum to a velocity scale, weighting the lines according to their depth and Landé factors, and essentially averaging the $I(v)$ and $V(v)$ signals from all these lines together. An example of how this technique can enhance SNR is seen by comparing the directly observed spectrum of the cluster magnetic Ap star HD 317857 in the region of the Fe II line λ4923 in Figs. 6, where the Zeeman signature in the strong line is hardly visible

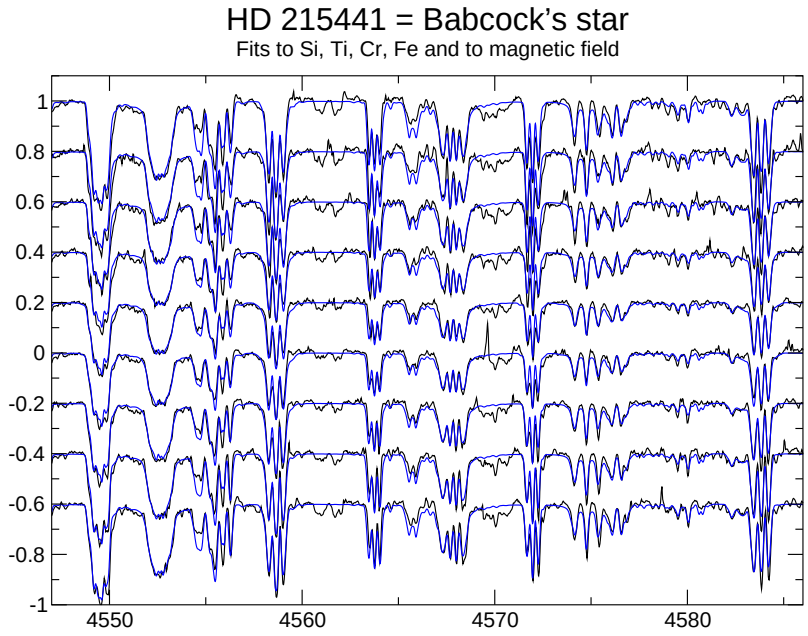

Fig. 3. An example of a fit to a (strongly!) magnetic star, HD 215441 = Babcock's star, which has a mean field modulus $\langle B \rangle$ of about 34 kG. The field (a colinear dipole, quadrupole, and octupole) is adjusted to give the correct line splitting and the measured longitudinal field $\langle B_z \rangle$. Abundances are fit on three rings concentric about the magnetic axis. Observations are in black, model spectra are in blue. The successive spectra are approximately evenly spaced through the 9.5-day rotation period of the star.

above the noise, with the LSD profile of Fig. 6, where the signature is very clean and strong.

An extremely attractive feature of LSD V profiles is that, because the form of the V profile of single lines is very similar from line to line, they can be modelled almost as if they were real single spectral lines. Unfortunately, this is not true of Q and U profiles.

The LSD technique may be used to detect really weak fields. For example, although the very bright star HD 112185 = ϵ UMa is clearly a magnetic Ap star, its field is very weak ($\langle B_z \rangle \sim 10^2$ G) and was for a long time very difficult to detect. With the MuSiCoS polarimeter the field was easily measured with high SNR (Wade et al. 2000b).

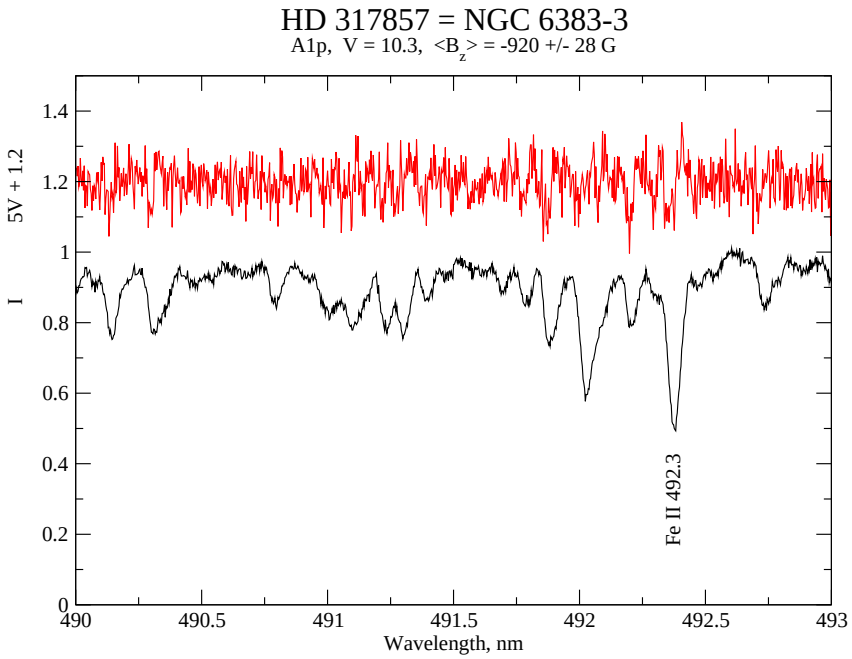

Fig. 4. A section of the spectrum of the magnetic Ap cluster member HD 317857 showing $I(\lambda)$ below and $V(\lambda)$ above.

7 Recent advances in modelling magnetic fields of Ap stars

With the newly available four-Stokes parameter spectra, we can return to the problem of modelling magnetic stars. A first interesting question is to test how well the models derived from simple field measurements ($\langle B_z \rangle$ or other field moments) and I spectra by comparing observed $[I, Q, U, V]$ spectra obtained at a series of rotational phases with the predicted line profiles obtained using the model derived from moments. This has been done for simple models of the magnetic Ap stars HD 137909 = β CrB and for HD 65339 = 53 Cam. In both cases the models obtained using only I spectra and field moments such as $\langle B_z \rangle$ yield only rather disappointingly imprecise fits, especially to Q and U profiles. The conclusion is that simple field models from field average measurements, although they probably capture reasonably well the gross structure of a field, provide at best only a first, rather rough, approximation to the detailed structure of the magnetic field (Bagnulo et al. 2001).

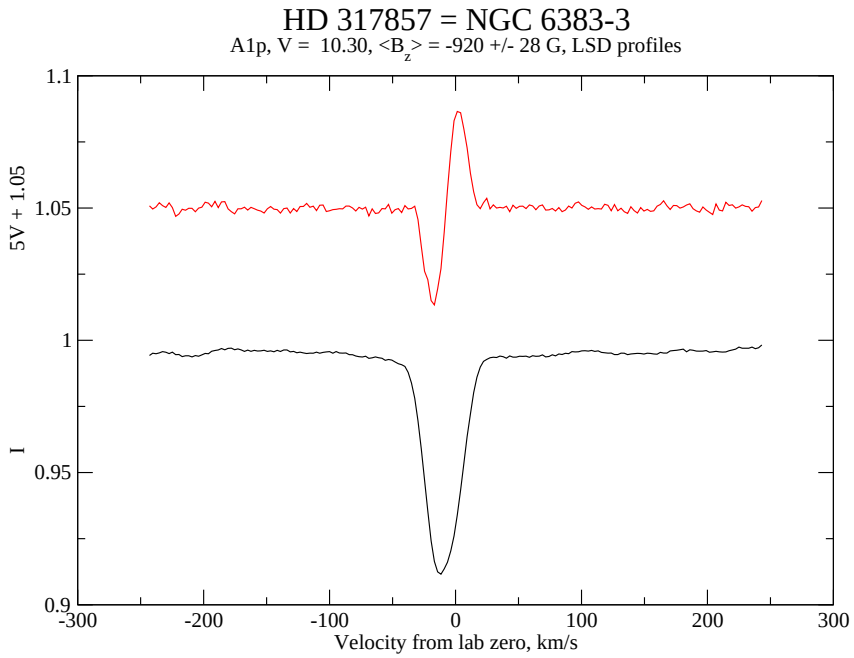

Fig. 5. The LSD spectrum of an ESPaDOnS spectrum of HD 317857 based on more than 4800 lines.

The solution is clearly to develop a mapping code that iteratively improves a map of the field and chemical abundances over the stellar surface by testing the map against spectra of all four Stokes parameters at many rotational phases. This process requires many cycles of forward computation of Stokes parameters from the current approximate map, comparison with observed Stokes parameters, adjustment of the approximate map (at many stellar surface "pixels") in such a way that the agreement of predicted and observed spectra are expected to improve. The process is carried through many cycles until convergence is achieved.

This programme has been carried out for a small number of stars using a computer code developed by N. Piskunov and O. Kochukhov. A very impressive result has been obtained for the star 53 Cam, for which MuSiCoS spectra for all four Stokes parameters were obtained at about 20 phases. Maps of the magnetic field structure and of the large-scale distribution of Si, Ca, Ti, Fe and Nd have been obtained which predict spectra that are in extremely impressive agreement with the observed spectra (Kochukhov et al. 2004). In the case of this star, it is

found that the global structure of the field and abundance distributions resemble those derived using the simple schematic model capabilities of Zeeman (Landstreet 1988), but the new maps reveal a degree of surface complexity that simply cannot be captured with techniques that adjust only a few modelling parameters.

A code for mapping magnetic fields on cool active stars on the basis of spectropolarimetry has been developed by J.-F. Donati. For such stars, the mapping problems are somewhat different than for magnetic Ap stars. The cool stars do not have abundance variations over the surface, but they do have surface temperature variations. Furthermore, the fields of the cool stars are relatively weak, and they cover only a fraction of the stellar surface. As a result the polarisation in Q and U is undetectable at present, and that in V is only available with adequate SNR if many lines are combined using LSD. Naturally, maps derived using only LSD I and V spectropolarimetry are significantly less well-constrained than those based on four Stokes parameters of individual spectral lines. Nevertheless, impressive results have been achieved for a number of such stars as well, for example for the RS CVn star HR 1099 by Petit et al. (2004).

Thus we see that computation of spectra of magnetic stars using a good underlying physical model is quite practical, and with sophisticated mapping techniques such spectrum synthesis is beginning to yield detailed maps of both hot (well, tepid) and cool magnetic stars. The first such maps strongly reinforce the impression that magnetic Ap stars have fields that are structurally quite different from those of active cool, solar-like stars.

References

Bagnulo, S., Wade, G. A., Donati, J.-F., Landstreet, J. D., Leone, F., Monin, D. & Stift, M. J. 2001, A&A 369, 889

Born, M. & Wolf, E. 1965, *Principles of Optics*, (London: Pergamon Press), Sec. 10.8.3

del Toro Iniesta, J. C. 2003, *Introduction to Spectropolarimetry* (Cambridge: Cambridge University Press)

Donati, J.-F., Semel, M., Carter, B. D., Rees, D. E. & Cameron, A. C. 1997, MNRAS 291, 658

Hauscildt, P. H. & Baron 1999, J. Comput. Appl. Math. 109, 41

Khan, S. & Shulyak, D. 2007, A&A 469, 1083

Kochukhov, O., Bagnulo, S., Wade, G. A. et al. 2004, A&A 414, 613

Kurucz, R. L. 1979, ApJS 40, 1

Landstreet, J. D. 1988, ApJ 326, 967

Landstreet, J. D., Barker, P. K., Bohlender, D. A., & Jewison, M. S. 1989, ApJ 344, 876

Martin, B. & Wickramasinghe, D. T. 1979, MNRAS 189, 883

Novotny, E. 1973, *Introduction to Stellar Atmospheres and Interiors*, (London: Oxford University Press), Part 2

Petit, P., Donati, J.-F., Wade, G. A. et al. 2004, MNRAS 348, 1175

Shurcliff, W. A. 1966, *Polarized Light*, (Cambridge, USA: Harvard University Press), Sec. 2.3

Stenflo, J.-O. 1994, *Solar Magnetic Fields: Polarized Radiation Diagnostics* (Berlin: Springer)

Unno, W. 1956, Pub. Astr. Soc. Japan 8, 108

Wade, G. A., Donati, J.-F., Landstreet, J. D. & Shorlin, S. L. S. 2000, MNRAS 313, 823

Wade, G. A., Donati, J.-F., Landstreet, J. D. & Shorlin, S. L. S. 2000, MNRAS 313, 851