Exercises for Leverhulme Lectures on Stellar Magnetism

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1 Overview of stellar magnetism

Note: since most astronomers interested in stellar magnetism use the Gaussian cgs system of units rather than the SI system (and so papers generally report observations in these units), all the exercises below are in the cgs system of units. You are of course free to do all exercise in your favourite system of units. Values of physical constants can easily be found on the Web, especially in SI units.

1. Convert a field strength of 1 T (Tesla, the SI unit of field strength) to G (Gauss, the unit of field strength in Gaussian cgs units). Look up the (polar) magnetic field strength of the Earth in G.
2. In the atmosphere of a main sequence A star, a typical total particle number density near optical depth unity would be around 10^{15} particles cm^{-3}, at a temperature of around 9000 K. Estimate the thermal energy density in the gas. Now suppose that a field of 1000 G permeates this atmosphere. What is the energy density (given by $B^2/8\pi$ in cgs Gaussian units) in the magnetic field? Compare these two numbers. Does this comparison suggest any obvious consequences to you?
3. In a dense gas cloud that might collapse to become a star, the particle number density could be of order 10^4 cm^{-3} and the temperature might be 20 K. Observed magnetic fields in such clouds are usually of the order of a few μG. Compare the energy densities in the gas and in the magnetic field.
4. In the deep interior of a main sequence star, the density is of the order of 10^{24} particles cm^{-3}, and the temperature is of order 10^7 K. How strong would a magnetic field have to be in order to have an energy density similar to that of the gas? (It is doubtful that the interior fields are this large, but this is not known with certainty.)
5. Solar flares are thought to be powered by the abrupt conversion of magnetic energy into thermal energy, leading locally to an explosion. Suppose a flare occurs in a region in which the number density of particles is of order 10^{15} cm^{-3}, and the local temperature is 5000 K. If the energy density of a field of 3000 G is converted "rapidly" to thermal energy, what is the maximum temperature to which the gas might be heated? Does this seem to be an energetically plausible explanation for a flare?

2 Review of atomic physics and the Zeeman effect

1. In an atomic Hamiltonian, the size of various terms can be estimated by setting a typical length equal to the Bohr radius a_0, any angular momentum value to $\hbar$, and the electrostatic potential to Ze^2/r. Use these approximations to estimate the relative order of magnitude of the various terms in the magnetic Hamiltonian discussed in class, and show that the magnetic terms are small compared to the non-magnetic terms for a field of 1 kG. Also estimate the field strength that would be needed for the linear magnetic term to be of the same order of magnitude as

the dominant terms. (Since the Schroedinger equation is linear and homogeneous in the wave function, its dimensions do not matter.)

2. Compute the actual Zeeman pattern for the Fe II spectral line at 4273 Å. The transition is $^4P_{3/2}$ to $^4D_{1/2}$. How many π components, and how many σ^+ components, are expected? Remember to respect the selection rules for the transitions between sub-levels.

3. Compare the size of the Zeeman splitting at 5000 Å (0.5 μm) to the splitting from a transition between the same spectroscopic terms at 2 μm.

4. Find the terms involved in some transition which has no Zeeman splitting at all (a "magnetic null line").

5. Show that in the Paschen-Back limit a magnetically split line shows only one π component and two σ components.

6. Suppose you have a spectrum of a star whose atmosphere is at about 10 000 K, which shows thermal line broadening but no rotational broadening at all. Suppose your spectrum is obtained with a spectrograph with resolving power $\lambda/\Delta\lambda = 50\,000$. Estimate the line width due to thermal motions, and due to instrumental broadening, and combine these two in quadrature to obtain an estimate of total broadening. How large would a magnetic field have to be (order of magnitude) to show clearly separated π and σ components? How large would the field have to be if $v \sin i = 10$ km s^{-1}?

7. Find a Web site discussing the solar cycle with a plot of the "sunspot number" as a function of time over at least the past two sunspot cycles (the past 22 yr). Where are we in the current solar cycle? Is anything odd going on?

3 Measuring stellar fields: polarimeters and Stokes parameters

1. Consider linearly polarised light travelling in a ray with the direction of polarisation vertical. Suppose that this light passes through a half-wave plate (HWP) with its fast axis (the axis along which the light travels with a higher speed) vertical and its slow axis horizontal. By resolving the polarisation direction of the light beam along the fast and slow axes of the HWP and considering that the output beams along these two axes suffer a quarter wave relative phase change, show what the polarisation nature of the light beam is after passing through the wave plate. Then rotate the wave plate so that its fast axis makes a 45° angle with the vertical, and repeat this exercise.

2. Suppose that the HWP or the preceding exercise is oriented at 45° to the vertical and followed by a polaroid whose orientation is such that light vertically polarised is transmitted and horizontally polarised light is extinguished. If the light beam is initially horizontally polarised, what fraction of the light is still present after the polariser? Answer the same question for light initially vertically polarised.

3. Go through similar reasoning to the preceding problems to convince yourself that a quarter-wave plate followed by a polariser with its transmission axis orientated at 45° to the fast axis of the wave plate will pass one sense of circular polarisation and extinguish the other, and thus show that this combination is effectively a polariser for circular polarisation.

4. A circular analyser is used to measure the intensity of left circular and right circular polarisation through an isolated stellar spectral line at 6149.25 Å. The two measurements are normalised so that the Stokes parameter I is equal to 1.00 in the continuum. Near the spectral line, the intensity in left circularly polarised light is described approximately by $I_L(\lambda) = 0.5 - 0.21\exp(-(\lambda - \lambda_{L0})^2/(\Delta\lambda)^2)$ and the intensity in right circularly polarised light is $I_R(\lambda) =$

$0.5 - 0.21\exp(-(\lambda - \lambda_{R0})^2/(\Delta\lambda)^2)$, where $\Delta\lambda = 0.033$ Å, $\lambda_{L0} = 6149.235$ Å, and $\lambda_{R0} = 6149.265$ Å. Calculate Stokes I and Stokes V over about 0.5 Å through this spectral line. Does the I parameter behave about as you would expect for a spectral line?

4 Observations of magnetic fields in stars

1. Suppose that we are able to measure the flux F (integrated over all wavelengths) from a star in physical units such as erg cm^{-2} s^{-1}, and that we know the distance d to the star from parallax measurements. Derive an expression for the luminosity L from F and d. If the effective temperature T_e has been determined from the energy distribution of the stellar flux, one can determine the stellar radius using $L = 4\pi R^2 \sigma T_e^4$, where σ is the Stefan-Boltzmann constant $5.67\,10{-}5$ ergs cm^{-2} s^{-1} K^{-4} in cgs units. Convert this expression to solar units using $T_e = 5770$ K for the Sun.

2. The magnetic Ap star HD 108945 has $F = 16.1\,10^{-8}$ ergs cm^{-2} s^{-1} and parallax $\pi = 12.09 \pm 0.27$ milli-arc sec. Find the luminosity L, including an estimate of the uncertainty arising from the uncertaint distance. Convert to solar units using $L_\odot = 3.86\,10^{33}$ erg s^{-1}. Assuming that $T_e = 8800$ K, find the stellar radius, which should be approximately $2.5 R_\odot$

3. Suppose that a star has a radius R, equatorial velocity v_e, and rotation period P. From simple geometry find the relationship between these three variables.

4. From spectral line broadening we can measure the projection $v_e \sin i$ of the equatorial rotation velocity along the line of sight, where i is the inclination of the line of sight to the stellar rotation axis. Use the relationship from the exercise above to find an expression for $\sin i$ assuming you know P, R and $v_e \sin i$. For HD 108945 (see exercises above) we know that $P = 2.004$ d and that $v_e \sin i = 65$ km s^{-1}. Estimate i.

5. Imagine that a magnetic Ap star has a magnetic field whose distribution over the stellar surface is roughly that of a magnetic dipole. Suppose that the axis of the dipole is inclined to the stellar rotation axis by 45°, and that the line of sight towards the star also makes an angle of 45° with the rotation axis. Suppose that the maximum field measured during a stellar rotation is 1200 G. Sketch qualitatively how you would expect the observed field strength to vary during one rotation period.

5 Radiative transfer and polarised light

1. Use the approximate analytical expression derived in class for the variation of specific intensity $I_\nu(0,\omega)$ with frequency to explore how a local spectral line profile varies as the strength of the line increases. Assume that the form of the line absorption profile is

$$\eta_\nu = \eta_0 \exp(-(\nu - \nu_0)^2/\Delta\nu^2), \qquad (1)$$

   where $\Delta\nu = \xi_0 \nu_0 / c$ and $\xi_0 = (2kT/m)^{1/2}$ is the most probable thermal velocity. Use your favourite plotting routine to plot the line profile for $\mu = 0.8$ and 0.2, $\beta = 2$ and $\eta_0 = 0.1, 1, 10, 100, 1000$. Explain why the line strength grows much more slowly for large η_0 than for small η_0, and why the profile is different for large μ than small μ.

2. You will probably recall from previous courses on stellar atmospheres that strong lines develop broad wings (like those of the H Balmer lines or the Ca II H and K lines). In the calculation above, these do not appear. Why?

3. Use the analytical expression for specific intensity to predict how the *continuum* intensity varies with μ from centre to limb of a star with the parameters of the first problem. Plot the resulting behaviour.

4. Write a very simple computer routine to numerically integrate the differential form of the equation of transfer for I_c from deep in the atmosphere (say $\tau_c = 10$) to the surface. Use a step size of no more than 0.1 in τ_c. Compare the run of I_c from your programme with the analytical expression for the linear source function case. Could you use the same step size (0.1 in τ_c) within a spectral line in the more general case of $\eta_\nu = \eta_\nu(\tau_c)$?